I. The results obtained using the extrapolation method are in reasonable agreement with directly measured values of σ_{np} shown in Fig. 5, when the experimental uncertainties are taken into account. The forms of the extrapolation functions are the same for both energy regions.

An inescapable conclusion to be drawn is that the method is hard to apply because of the difficulty of accumulating sufficient data to establish each value of $\partial^2\sigma/\partial(p^2)\partial(w^2)$ to good statistical accuracy and because of the uncertainty about the form that the extrapolation function should take, a point on which the theory has nothing to say. Nevertheless, we believe that we have found empirically a method for making the extrapolation correctly for the *pn* system, and that the same procedures can be used to measure σ_{nn} by the Chew-Low method.

ACKNOWLEDGMENTS

It is a pleasure to express our thanks to all the people who have helped in carrying out this experiment: to the emulsion scanners at Brown University, in particular to Mrs. Dorothy Stepak, for much careful work in scanning and measuring the tracks; to John Stevens and Jewel Poris for assistance in data processing; and to Robert J. Schneider and the staff of the Harvard Cyclotron for help in exposing the nuclear emulsions. We are very grateful to G. Breit and R. E. Seamon for supplying the cross sections calculated from the Yale phase-shift analysis.

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Inelastic Nuclear Interactions of High-Energy Electrons and Muons in Emulsion*

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An experiment has been carried out to examine the inelastic nuclear interaction properties of high-energy electrons and muons in emulsion. A study was made of interactions produced by 10.5-GeV muons and 10.0- and 16.0-GeV electrons. Total and differential cross sections have been measured and compared with calculations carried out with the formulation of Hand and Wilson for inelastic lepton scattering. On the basis of the calculations, the ratio of electron to muon total cross sections was found to be 3.5:1, whereas experimentally the ratio was measured to be 2.18 ± 0.40 for the 10-GeV leptons. To explain the total cross section for electrons, an average scalar-photon contribution equal to 12% of the transverse-photon cross section is needed, which is consistent with other experiments. The muon cross section was found to be significantly larger than expected, and it cannot be explained by a similar scalar-photon contribution. The energy dependence of the total cross sections is in agreement with theory, as are the angular distributions of the scattered leptons.

I. INTRODUCTION

THE inelastic nuclear scattering of electrons and muons provides direct information on the applicability of quantum electrodynamics. It also serves to examine any possible differences in the nature of the interactions for the electron and muon. In order to investigate these two questions, an experiment has been carried out with photographic emulsions exposed to 10.0- and 16.0-GeV electrons at the Stanford Linear Accelerator and 10.5-GeV muons at the Brookhaven Alternating Gradient Synchrotron. In addition, previous muon exposures at 2.5 and 5.0 GeV have been reexamined to supply a comprehensive analysis of the inelastic lepton interaction over a wide range of energies.

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Experiments on the investigation of meson production by electrons were first carried out by Panofsky, Woodward, and Yodh.[1] In a subsequent experiment by Panofsky and Allton,[2] the electroproduction of the $N^*(1238)$ resonance was examined. Later experiments have investigated the structure of the $N^*(1238)$ and higher resonances.[3–5] The contribution of longitudinal photons to inelastic electron processes has recently been observed for a few values of energy and momentum transfer.[5,6]

Inelastic muon interactions were first studied using nuclear emulsions and underground cosmic rays.[7] It became possible to produce muon beams with small contamination using accelerators and nuclear emulsion experiments have measured inelastic cross sections for the energies available.[8–10]

II. THEORY

The differential cross section for inelastic lepton nucleon scattering, including the lepton mass, has been calculated by Hand and Wilson,[11] for the exchange of a single photon (Fig. 1) and is given by

$$d^2\sigma/d\Omega dE' = \alpha p'^2 K/4\pi^2 p E' q^2$$

$$\times\left[[2+\cot^2(\tfrac{1}{2}\theta^*)]\sigma_t+\left(\frac{4m^2}{q^2}+\cot^2(\tfrac{1}{2}\theta^*)\right)\sigma_0\right]. \quad (1)$$

In this expression, θ^* is the scattering angle of the lepton in the Breit frame of the target nucleon, m the mass of the lepton, and $\sigma_t(q^2,K)$ and $\sigma_0(q^2,K)$ are the transverse- and scalar-production cross sections for virtual photons. The term K is given by $K=q_0-q^2/2M$ where M is the nucleon mass and $q^2=\mathbf{q}^2-q_0{}^2$ is the square of the four-momentum transfer. To express $\cot^2(\frac{1}{2}\theta^*)$ in terms of laboratory quantities, the component of incident lepton momentum perpendicular to q is divided by $q^2/4$, and evaluated in both the lab frame and the nucleon Breit frame. Making no approximations, one has

$$\cot^2(\tfrac{1}{2}\theta^*)=4p^2p'^2\sin^2\theta/q^2(q^2+q_0{}^2). \quad (2)$$

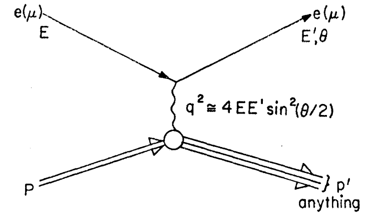

FIG. 1. First-order graph for inelastic scattering of electrons (muons) with kinematic variables.

With this expression for $\cot^2(\frac{1}{2}\theta^*)$ substituted into Eq. (1), one has the differential cross section, including the lepton mass.

In an experiment of this type, the total cross section for inelastic electron (muon) scattering, and the angular distribution of the scattered leptons can be measured, but the final energy of the scattered lepton cannot be determined with any precision. It is therefore necessary to integrate Eq. (1) over the energy of the outgoing electron (muon). An exact calculation would require detailed knowledge of the scalar and transverse photon cross sections for all values of q^2 and K. Since such data are quite incomplete, it is necessary to make the approximation suggested by Hand and Wilson[11]:

$$\sigma_t(q^2,K)=\sigma_\gamma(K)F^2(q^2), \quad (3)$$

where $\sigma_\gamma(K)$ is the photoproduction cross section for real photons of energy K, and $F(q^2)$ is taken to be the nucleon form factor, with the expression

$$F(q^2)=(1+q^2/0.71)^{-2}\ (q^2 \text{ in GeV}^2) \quad (4)$$

taken as the best fit to the form factor data. With these approximations, the expression for the differential cross section becomes:

$$\frac{d^2\sigma}{d\Omega dE'}=\frac{\alpha K p'^2\sigma_\gamma(K)F^2(q^2)}{4\pi^2 EE'}\left[\left(2+\frac{4E^2p'^2\sin^2\theta}{q^2(q_0{}^2+q^2)}\right)\right.$$

$$\left.+\left(\frac{4m^2}{q^2}+\frac{4E^2p'^2\sin^2\theta}{q^2(q_0{}^2+q^2)}\right)\frac{\sigma_0(q^2,K)}{\sigma_t(q^2,K)}\right]. \quad (5)$$

It is necessary to include the photon cross section from photoproduction experiments. For energies below 5 GeV, the compilation of photon cross sections from Cone *et al*,[4] was used. For photon energies greater than 5 GeV, it was assumed that the photoproduction cross section equals 100 μb, following the example of Friedman and Kendall.[12]

With the above approximations, an integration was carried out over the final energy of the scattered electron (muon), giving the angular distribution of the scattered lepton. A second integration was then made over all scattering angles, to obtain the total production cross

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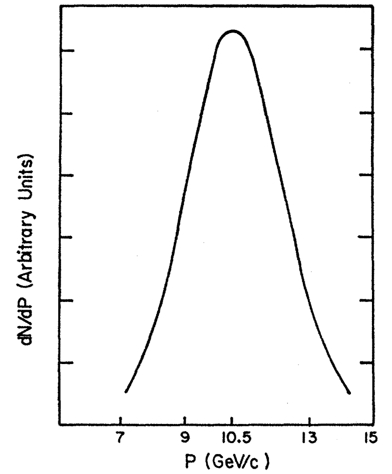

FIG. 2. Momentum distribution of the Brookhaven 10.5-GeV/c muon beam.

section. This approach assumes that the leptons interact with a single nucleon. The results of the calculation are included in the results and conclusions section of this paper, and are compared with the measured cross sections.

III. EXPERIMENTAL PROCEDURE

Stacks of Ilford G-5 nuclear emulsion were exposed to 10.0±0.03- and 16.046±0.007-GeV electrons at the Stanford Linear Accelerator, and to 10.5-GeV muons at the Brookhaven A.G.S., with the beam perpendicular to the emulsion surface. The momentum distribution in the muon beam is shown in Fig. 2. The pion contamination in the beam has been measured[13] and the ratio of pions to muons found to be less than 10^{-6}. At this level, the background from pion events is about 0.1%, much less than the statistical errors of the experiment. A description of the production of the muon beam, and its properties, has been described.[13]

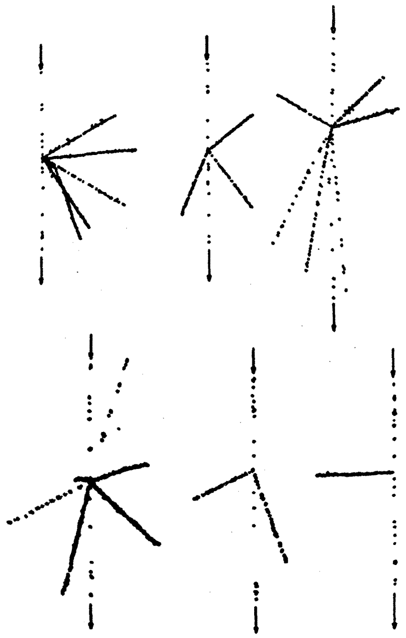

FIG. 3. Sample events. The six events shown here were found in area scanning a plate exposed to 16-GeV electrons, with the plane of the emulsion parallel to the electron beam. The arrows indicate the incident and the scattered electrons.

The processed emulsions were area scanned for all stars, with binocular microscopes equipped with 10× oculars and 22× oil-immersion objectives. Figure 3 shows six typical stars as they appear through the microscope. These particular stars were taken from a plate exposed with the beam parallel to the plane of the emulsion, and therefore the incident and scattered electron is visible and indicated with an arrow. For all stars with an incident beam track passing through the center, the scattering angle of the lepton was measured.

All events with more than two prongs were assumed to be due to meson production, as were those one-prong events with scattering angles greater than 0.3°. These one-prong events with smaller scattering angles were taken to be due to the nuclear giant resonance. This criteria has been used in previous experiments and has been shown[8] to be a good approximation.

The data were corrected for coincidences of cosmic ray produced stars with incident electrons (muons), and for scanning efficiencies.

IV. RESULTS

Area scanning of the emulsions resulted in finding 447 inelastic nuclear interaction events for 16-GeV electrons, 348 events for 10-GeV electrons, and 87 events for the 10.5-GeV muons, which appeared to have been produced by the incident leptons. In the case of the electrons, only events in about the first 0.1 radiation length of the emulsion, including paper covering, were considered. This precaution reduced to negligible proportions the effects due to a bremsstrahlung degraded electron beam. The normalized prong distribution for these events is given in Fig. 4 along with previous results[8] at lower muon energy. After subtracting cosmic ray induced background events, nuclear giant resonance events, and correcting for scanner efficiencies, the total cross sections listed in Table I were obtained. In this table, σ_1 is the total theoretical cross section obtained by letting $\sigma_0 = 0$ and σ_2 is the result when σ_0 is set equal to σ_t in the integration of Eq. (5). All cross sections are given in μb per nucleon of the emulsion. The energy dependence of the muon and electron cross sections is given in Fig. 5. It is to be noted on this figure that three lines are drawn; the lower one corresponding to the calculated σ_1, the upper one for σ_2, and the intermediate curve corresponds to the cross section obtained by letting

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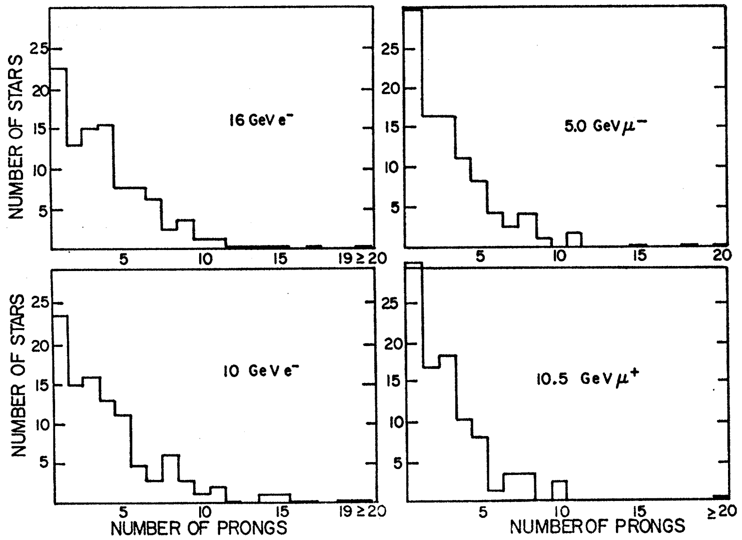

FIG. 4. The four histograms give the prong distribution for 10- and 16-GeV electrons, and for 5- and 10.5-GeV muons. The 5-GeV muon data are taken from Ref. 9.

$\sigma_0=0.3\sigma_t$. A contribution of 30% or less due to σ_0 has been observed in previous experiments.[3] The error limits are one standard deviation, and are only the statistical ones. Errors other than statistical, which can enter in, are uncertainties in the scanning efficiency, the emulsion thickness and density, and in the determination of the flux. The upper limit of the error due to these factors is estimated to be 7%, and is common to both muon and electron measurements. In the case of the 10-GeV electrons and muons where a very accurate relative measurement was desired, the scanning and measuring were done by the same person. The scanning efficiencies were checked by independent scanning, and found to be $(92\pm 2)\%$, and the uncertainty in scanning efficiency is included in the overall 7% max. systematic error.

The results of this experiment are in basic agreement with the predictions of Hand and Wislon,[11] considering the uncertainty of the nature of the contribution of σ_0. In Table I, it is apparent that the measured total cross sections are all too large to be accounted for by the transverse photon cross section only. In order to explain the total cross sections, it is necessary to include a contribution due to the scalar photon cross section, which amounts to an average of about 12% of the transverse cross section, for both 10- and 16-GeV electrons. The ratio of σ_0/σ_t has been measured at a few points,[5,6] and is seen to vary rapidly with q^2. For example, σ_0/σ_t varies from about 0.3 at $q^2=0.3$ GeV^2, to 0.0 at $q^2=0.4$, for K fixed at 1238 MeV.[5] Thus, the value $\sigma_0/\sigma_t=0.12$ from this experiment, is only a rough estimate of the contribution of σ_0 averaged over all energies, and is con-

TABLE I. Measured and theoretical inelastic nuclear cross sections for electrons and muons. The data for the 10.5-GeV muons is very nearly the same as that reported earlier (p. 520 of Ref. 5), but with limited statistics.

Energy and particle	Cross section per nucleon (μb) Theoretical σ_1 $\sigma_0=0$	σ_2 $\sigma_0=\sigma_t$	Measured σ
10.0 GeV, e^-	21.7	43.9	23.6±1.4
16.0 GeV, e^-	32.4	65.4	37.3±2.0
10.5 GeV, μ^+	6.1	13.3	11.4±1.4

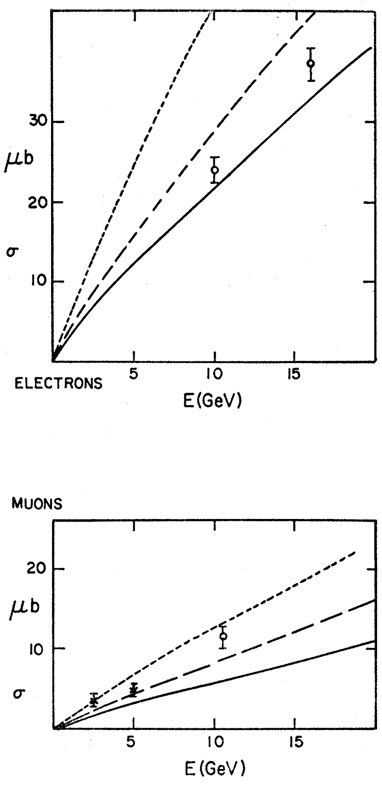

FIG. 5. Total cross sections for meson production by muons and electrons, as a function of the primary lepton energy. The calculated curves represent the following conditions: $\sigma_0=\sigma_t$ (short dashes); $\sigma_0=0.3\times\sigma_t$ (long dashes); $\sigma_0=0$ (solid line). The data indicated by × are from Ref. 8.

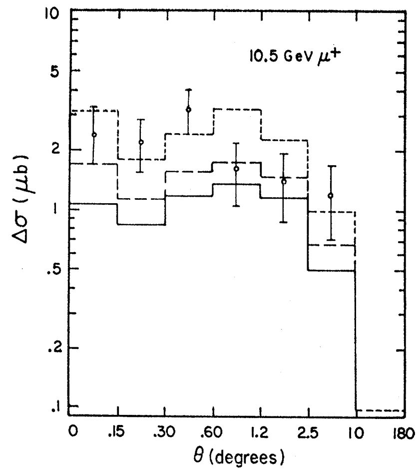

FIG. 6. Total cross sections in the angular intervals $\Delta\theta$ for 10.5-GeV positive muons. The calculated histograms represent the following conditions: $\sigma_0=\sigma_t$ (short dashes); $\sigma_0=0.3\times\sigma_t$ (long dashes); $\sigma_0=0$ (solid line).

sistent with the values mentioned above, from other experiments.

The data on 10.5-GeV muons in Table I are of lower statistical accuracy than that for the electrons, and are also somewhat uncertain due to the momentum spread in the beam, Fig. 2. However, if one assumes a similar 12% average contribution from scalar photons for the muon interactions, then the measured total cross section is 4.4 μb too large, which amounts to 3.1 standard deviations on the statistical error. This also is consistent with the data at 2.5- and 5.0-GeV shown in Fig. 5. It is not meant to imply that the muons actually have a larger scalar contribution, but rather that the same average scalar contribution will not explain the total cross section for the muons.

Theoretical calculations[11] indicated that the ratio of electron to muon total cross sections should be about 5:1. The original calculations omitted the effect of the photon cross sections, and the nucleon form factor. On the basis of the more detailed computer calculation with Eq. (5), the ratio was found to be 3.7:1 for $\sigma_0=0$, or 3.5:1 for $\sigma_0=\sigma_t$. This ratio is seen to be rather insensitive to the average scalar photon contribution, and is also not strongly dependent upon the other assumptions made in the calculations. The experimental results give a ratio of 2.18 ± 0.40 for 10.0-GeV leptons. To compute this experimental ratio, the experimental cross section for the muons was reduced by the factor 5.8/6.1, the ratio of the calculated cross sections at 10.0 and 10.5 GeV ($\sigma_0=0$), to take the difference in energy into account. This result is seen to be 3.5 standard deviations smaller than predicted, including statistical errors only. If the maximum systematic error of 7% is applied to both cross sections, the ratio can be increased to 2.49 ±0.46, which is still too small by 2.5 standard deviations. The experimental points for 2.5-GeV and 5.0-GeV muons from Ref. 8 on Fig. 5 are seen to agree with the general results of this experiment, concerning muon total cross sections.

The angular distribution of scattered muons of 10.5 GeV is shown in Fig. 6. The upper histogram corresponds to calculations from Eq. 5 with $\sigma_0=\sigma_t$, the middle one with $\sigma_0=0.3\sigma_t$, and the lower for $\sigma_0=0$. The corresponding histograms for the electrons are given in Figs. 7 and 8. The experimental points for the muons in Fig. 6 are more noticeably higher than the lower histogram in contrast to those for the electrons in Figs. 7 and 8.

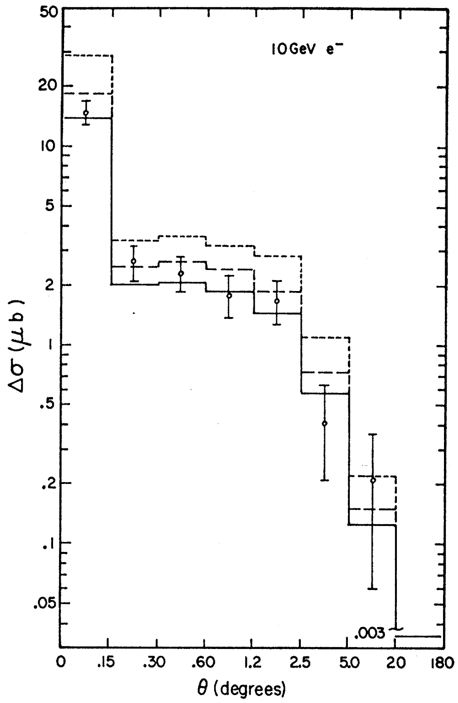

FIG. 7. Total cross sections in the angular intervals $\Delta\theta$ for 10.0-GeV electrons. The calculated histograms are for the same parameters as for Fig. 6.

The larger nature of the muon cross section as noted above for the total cross section when analyzed with Eq. 5, appears to be confined to all angles. The electron observations from Figs. 7 and 8 also indicate that a scalar contribution to the cross section is required for all angles.

The very noticeable differences in Figs. 6, 7, and 8 between the muon and electron histograms for angles from 0.0° to 0.15° is a kinematic effect which allows electrons to have a lower limit for their four-momentum transfer than the muons. The agreement with theory for these small-angle points indicates that there is not significant error in subtracting out background events which would, of course, appear to have no measurable deflection.

An approximation to the four-momentum transfer is given by $t_0=4E^2\sin^2(\frac{1}{2}\theta)$. While this expression is accurate at small angles, it will overestimate the momentum transfer at the larger angles. In this experiment, however, it will give a good estimate for the relative cross section of muons and electrons as a function of four-momentum transfer. The differential cross section $d\sigma/dt_0$ was calculated and is presented as a function of t_0 for the 10.5-GeV muons in Fig. 9. The upper solid curve in the figure corresponds to $\sigma_0=\sigma_t$ in the calculation of Eq. 5 and the lower to $\sigma_0=0$. The curves for the 10.0- and 16.0-GeV electrons are presented in Figs. 10 and 11. While the agreement between experiment and calculation is surprisingly good considering the above approximation for t_0, it is to be noted again that the muon cross sections are relatively higher than those for the electrons to four-momentum limits of about 1 GeV^2.

V. CONCLUSIONS

The final results of the experiment will now be summarized.

(1) A numerical integration of Eq. (5) leads to the prediction that the total cross section for meson production should increase linearly with increasing electron or muon energy. This prediction becomes strongly dependent upon the assumption of $\sigma_\gamma(K)=100$ μb/nucleon at very high energies, but is not so strongly dependent upon it at the energies under consideration. This linear increase in the total cross sections is found to be accurate at energies up to 16 GeV for electrons, and 10.5 GeV for muons.

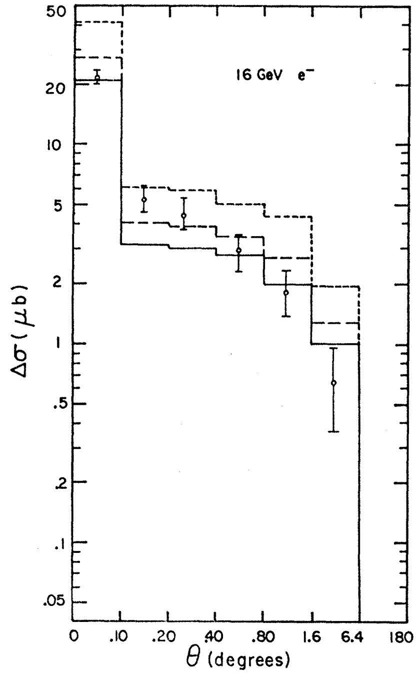

FIG. 8. Total cross sections in the angular intervals $\Delta\theta$ for 16.0-GeV electrons. The calculated histograms are for the same parameters as for Fig. 6.

(2) The total cross sections are all found to be larger than predicted by calculations which leave out the contribution of the scalar photon cross sections.

(3) In order to explain the total cross sections for the production of mesons by electrons, an average contribution of 12% of σ_t is needed for σ_0.

(4) In the case of the muons, the average contribution of the scalar photons would have to be much larger, 73%, to explain the total cross section.

(5) This effect is present at all angles although the statistical accuracy is poor for angles greater than 2.5°.

(6) Another way of stating this result is that the ratio of electron to muon cross sections at 10 GeV found to be 2.1 ± 0.5, contrasted to the approximate prediction of Hand and Wilson[11] of at least 5, and the more exact calculation of 3.5 from the use of Eq. (5).

Two questions arise concerning the large muon cross section. First, how accurate are the calculations. Several of the assumptions mentioned in Sec. II might be suspect in consideration of the magnitude of the results, although each assumption is felt to be a good approximation. The main difficulty is in the factorization of $\sigma_t(q^2,K)$ into $\sigma_\gamma(K)F^2(q^2)$. In the case of the resonances an additional multiplicative factor of $\mathbf{q}/K^2$ might be expected theoretically,[11] and has actually been observed[4,14] to hold approximately for small $\mathbf{q}^2$. The ob-

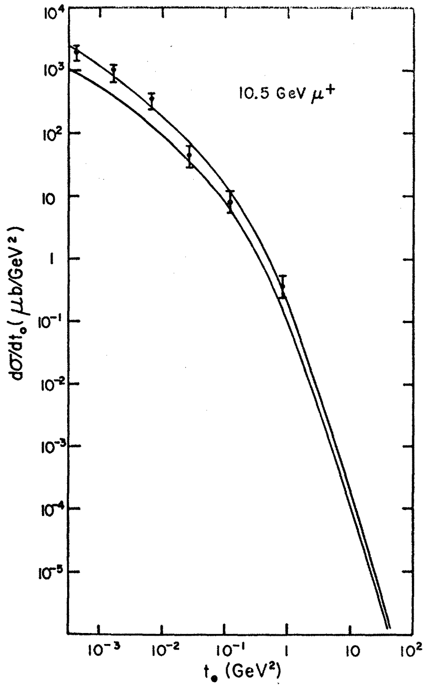

FIG. 9. $d\sigma/dt_0$ versus t_0 for 10.5-GeV positive muons. The two curves correspond to $\sigma_0=0$ (lower curve), and $\sigma_0=\sigma_t$ (upper curve).

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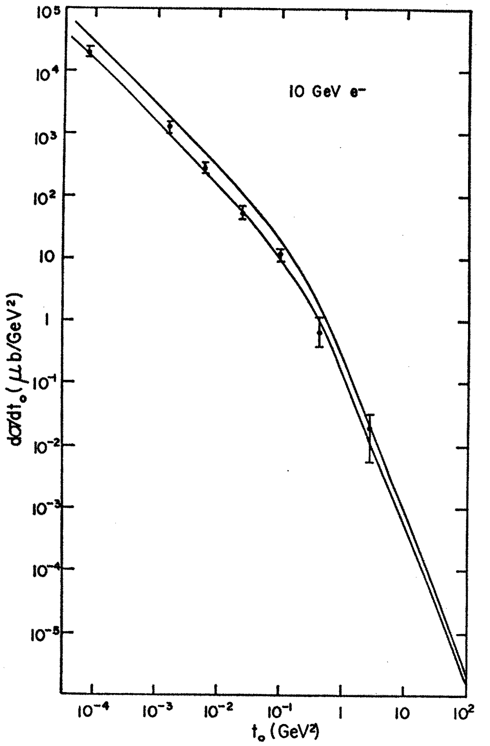

FIG. 10. $d\sigma/dt_0$ versus t_0 for 10.0-GeV electrons. The upper curve corresponds to $\sigma_0=\sigma_t$ and the lower to $\sigma_0=0$.

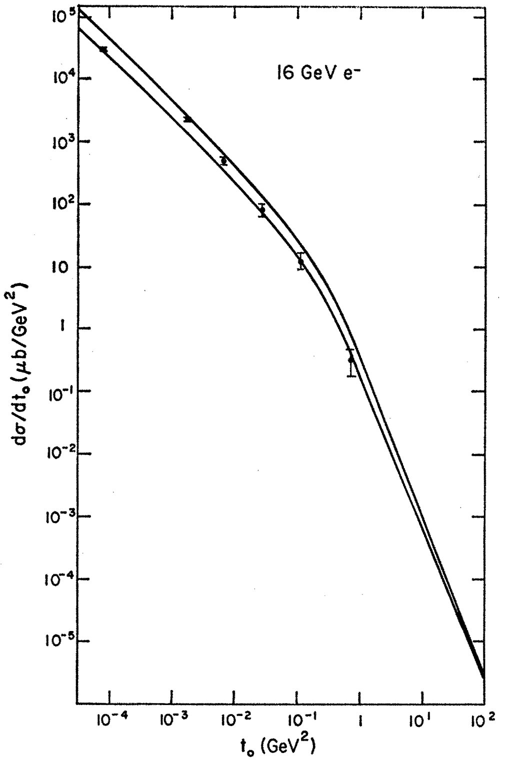

FIG. 11. $d\sigma/dt_0$ versus t_0 for 16.0-GeV electrons. The upper curve corresponds to $\sigma_0=\sigma_t$ and the lower to $\sigma_0=0$.

servations show that for energies K in the vicinity of a resonance a factor of $\mathbf{q}^n$ holds with specific values of n lying between 1.5 and 3.5. The factor $\mathbf{q}^n$ was inserted in Eq. (5) and with the best[4,14] empirical values of n, a machine integration was carried out. The ratio of the electron to muon total cross section was basically unaltered by including the above factor, but each calculated total cross section was increased by roughly 4%.

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