Inelastic Muon Interactions in Nuclear Emulsion at 2.5 and 5.0 GeV (*).

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Summary. — The inelastic scattering of high-energy muons from emulsion nuclei was measured with the use of a monoenergetic beam of 2.5 GeV/c and 5.0 GeV/c muons produced at the Brookhaven Alternating Gradient Synchrotron. Ilford G-5 stacks were exposed perpendicularly to the beam and Ilford K-5 stacks were exposed parallel to the beam. Area scanning resulted in a total of 135 events with four-momentum transfers greater than 26 MeV/c. The observed scattering distribution which extends up to 10 degrees, or a momentum transfer of about 900 MeV/c, is in good agreement with the predictions of the electromagnetic theory assuming single-photon exchange. There were also observed four events with scattering angles greater than 10 degrees which are not inconsistent with the theory. Pion production was observed with the major contribution coming from the $\frac{3}{2}$, $\frac{3}{2}$ resonance. The cross-sections for this process were found to be (3.6 ± 0.7) and (5.1 ± 0.5) microbarn for muon momenta of 2.5 and 5.0 GeV/c, respectively. Events involving much lower energy exchanges were found to have much larger cross-sections and were consistent with « giant resonance » photodisintegration observations.

1. – Introduction.

Many experiments concerning inelastic muon scattering have been difficult to interpret due to low intensity of the muon beam and its possible

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contamination with pions and other particles. When a very pure muon beam of high intensity [1,2] was produced at the Brookhaven Alternating Gradient Synchrotron it became possible to carry out inelastic muon scattering experiments with a much higher degree of precision. It became possible to study the interactions in detail, identifying secondary and scattered particles and to measure their distributions of particle momentum and angle.

Although the previous experimental work on elastic scattering of muons is not related directly to the present experiment concerning inelastic scattering, it has been considered in order to allow a comparison of the relative magnitudes of the two processes. A comparison with inelastic electron scattering experiments is considered also since, with the exception of the mass difference of the electron and the muon, the interaction is most likely the same.

Extensive work has been done on the multiple scattering of cosmic ray muons. The small intensity of particles required the use of thick scattering materials which, along with the problems of particle and momentum determination, made the interpretation of the results difficult. A satisfactory theory must both account for the multiple scattering from atomic nuclei, with distributed nuclear charge shielded by orbital electrons, and the less frequent larger angle scattering taking place within the nucleus. The Molière theory [3] treated the nucleus as a point charge modified by the shielding effect of the atomic electrons. This theory was correct at small scattering angles but could be expected to over-estimate the cross-section at larger scattering angles. The Olbert theory [4] accounted for the finite nuclear size by limiting the single scattering to the maximum angle $\theta = \lambda/R$, where λ is the de Broglie wave length of the incoming muon and R is the radius of the target nucleus. The Olbert theory, however, underestimates the cross-sections at large angles. The distribution of charge within the nucleus is accounted for in the Cooper and Rainwater [5] theory. This theory has been found to be most nearly correct and predicts at larger scattering angles results somewhat between those of Molière and Olbert.

Much of the early experimental work gave results which agreed with theory at the smaller scattering angles but with an excess of scatterings at larger angles. Instead of following the Cooper and Rainwater theory as expected, the distribution for larger angles better fitted the Molière theory and, hence,

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(1) BNL Internal Report, PD-47 (1962).

(2) T. Yamanouchi, R. W. Ellsworth, A. C. Melissinos, J. H. Tinlot, L. M. Lederman, M. J. Tannenbaum, R. Cool, A. W. Maschke and L. Marshall: *Bull. Am. Phys. Soc.*, **10**, 79 (1965).

(3) G. Molière: *Zeits. Natur.*, **3** a, 78 (1948).

(4) S. Olbert: *Phys. Rev.*, **87**, 319 (1952).

(5) L. N. Cooper and J. Rainwater: *Phys. Rev.*, **97**, 492 (1955).

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was considered anomalous. A good summary of the cosmic-ray results has been given by WOLFENDALE (6).

The advent of intense accelerator pion beams offered the opportunity of constructing high-energy muon beams with which more accurate experiments could be performed. The first result was that of MASEK *et al.* (7) who observed the scattering of 2 GeV/c muons from lead and carbon. Their results followed very closely the Cooper and Rainwater theory up to momentum transfers of ~ 260 MeV/c and the carbon data agreed with the Drell-Schwartz (8) theory up to momentum transfers of ~ 400 MeV/c. The results from the lead target at large angles were 90 standard deviations away from the distribution predicted by the Molière theory. Using the results of the carbon data they placed an upper limit on an anomalous cross-section for momentum transfers up to 400 MeV/c of

$$\frac{d\sigma}{d\Omega_{anom}} < 1.5\cdot 10^{-29}\ \text{cm}^2/\text{sr}\cdot\text{nucleon}\,.$$

Following DRELL (8), a limit was placed on the muon form factor of

$$1/\Lambda < 0.58\ \text{fermi}\,.$$

Similar results were obtained at CERN by CITRON *et al.* (9). Experiments carried out in emulsion by KIM *et al.* (10) and OREAR *et al.* (11) confirm the same result.

Most of the recent experiments are in good agreement with results which can be calculated for a nucleus having an extended charge distribution. At higher momentum transfers the charge distribution of the nucleus becomes less important and the shape of the individual nucleons becomes apparent. The elastic scattering of muons from protons was first measured by MASEK *et al.* (12) using a 1.2 GeV/c muon beam. Angular tests for elasticity were made and agreement was found with the Rosenbluth cross-section to momentum transfers of 850 MeV/c when using the proton form factors of Hofstadter.

(6) A. W. WOLFENDALE: *Cosmic Rays* (New York, 1963).

(7) G. E. MASEK, L. D. HEGGIE, Y. B. KIM and R. W. WILLIAMS: *Phys. Rev.*, **122**, 937 (1961).

(8) S. D. DRELL and C. L. SCHWARTZ: *Phys. Rev.*, **112**, 568 (1958).

(9) A. CITRON, C. DELORME, D. FRIES, L. GOLDZAHL, J. HEINTZE, E. G. MICHAELIS, C. RICHARD and H. OVERAS: *Phys. Lett.*, **1**, 175 (1962).

(10) C. Y. KIM, S. KANEKO, Y. B. KIM, G. E. MASEK and R. W. WILLIAMS: *Phys. Rev.*, **122**, 1641 (1961).

(11) D. KOTELCHUCK, J. G. MCEWEN and J. OREAR: *Phys. Rev.*, **129**, 876 (1963).

(12) G. E. MASEK, T. E. EWART, J. P. TOUTONGHI and R. W. WILLIAMS: *Phys. Rev. Lett.*, **10**, 35 (1963).

A very recent experiment carried out at Brookhaven by TINLOT *et al.* [13] has extended this result to momentum transfers of 1 GeV/c.

The inelastic scattering of electrons from protons has been studied by PANOFSKY and ALLTON [14] and by OHLSEN [15] in a noncoincidence experiment. When corrections due to pure electromagnetic processes were made the measured distribution of scattered electrons agreed well with the theory of Fubini, Nambu and Wataghin [16].

A more recent experiment of the same kind has been done by HAND [17], who found good agreement with the theory of Fubini *et al.* [16] except at the highest momentum transfer, $q^2 = 12\ \text{fermi}^{-2}$, where the resonant peak was shifted to higher values of photon energy.

An electroproduction experiment in which the final state π^0 was observed in coincidence with the scattered electron, has been performed by JORBA *et al.* [18] and the results were found to be slightly lower than the prediction of the theory of Salin [19].

The inelastic scattering of muons has been measured using cosmic-ray muons underground. GEORGE and EVANS [20] exposed emulsions at various depths below the surface of the Earth sufficient to eliminate, by absorption, the nucleonic component of the cosmic radiation. In scanning 843 cm^3 of emulsion, 265 stars of three or more prongs were found, resulting in a calculated muon cross-section of $(4.6 \pm 0.05) \cdot 10^{-30}$ cm nucleon^{-1}. Stars with light tracks were termed showers and the cross-section corresponding to these events was $(1.7 \pm 0.3) \cdot 10^{-30}\ \text{cm}^2\ \text{nucleon}^{-1}$. The shower particles were not identified but assumed to be mostly pions.

Observations in emulsions of the inelastic scattering of 2.5 GeV/c muons from the Brookhaven accelerator have been made by BACHHUBER [21]. At 5.0 GeV/c similar experiments have been made by KIRK [22] and JAIN *et al.* [23,24]. The total muon cross-sections observed in the above experiments

(13) R. COOL, A MASCHKE, L. M. LEDERMAN, M. TANNENBAUM, R. ELLSWORTH, A. MELISSINOS, J. H. TINLOT and T. YAMANOUCHI: *Phys. Rev. Lett.*, **14**, 724 (1965).

(14) W. K. H. PANOFSKY and E. A. ALLTON: *Phys. Rev.*, **110**, 1155 (1958).

(15) G. G. OHLSEN: *Phys. Rev.*, **120**, 584 (1960).

(16) S. FUBINI, Y. NAMBU and V. WATAGHIN: *Phys. Rev.*, **111**, 329 (1958).

(17) L. N. HAND: *Phys. Rev.*, **129**, 1834 (1963).

(18) J. P. PEREZ Y JORBA, P. BOUNIN and J. CHOLLET: *Phys. Lett.*, **11**, 350 (1964).

(19) P. L. SALIN: *Nuovo Cimento*, **32**, 522 (1964).

(20) E. P. GEORGE and J. EVANS: *Proc. Phys. Soc.*, A **68**, 829 (1955).

(21) C. BACHUBER: private communication.

(22) J. A. KIRK, D. M. COTTRELL, J. J. LORD and R. J. PISERCHIO: *Bull. Am. Phys. Soc.*, **8**, 602 (1963).

(23) P. L. JAIN: *Bull. Am. Phys. Soc.*, **9**, 393 (1964).

(24) P. L. JAIN and P. J. MCNULTY: *Phys. Rev. Lett.*, **14**, 611 (1965).

are in qualitative agreement with those obtained in the underground cosmic-ray experiments.

2. – Experimental procedure.

Stacks of Ilford G-5 and K-5 emulsions were exposed to the muon beam at the Brookhaven Alternating Gradient Synchrotron. The experimental arrangement for the exposure is shown in Fig. 1. The beam, prepared by COOL

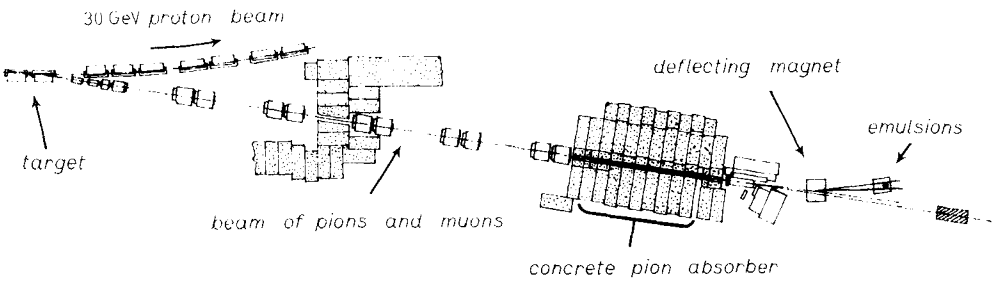

Fig. 1. – Experimental arrangement for muon beam scattering in emulsion.

et al. (2), was produced by allowing about 10% of the pions to decay into muons and then filtering out the remaining pions in a concrete absorber. As shown in Fig. 1, the intense beam of muons and pions was filtered through 32 feet of concrete. Based upon known interaction cross-sections, the pion contamination after the filter was found (2) to be less than one part out of 10^7. The particles were then subjected to momentum analysis in a deflecting magnet to produce a monochromatic beam. Ilford G-5 emulsions were exposed to muons of 2.5 GeV/c and 5 GV/c at an intensity of 10^6 muons/cm^2 with the plane of the emulsion perpendicular to the beam. At the same time, K-5 emulsions were exposed to muons of 5 GeV/c with an intensity of $4\cdot10^5$ muons/cm^2 and with the plane of the emulsion parallel to the beam. The momentum spread of the beam was calculated to be 10% and the measured angular spread is shown in Fig. 2.

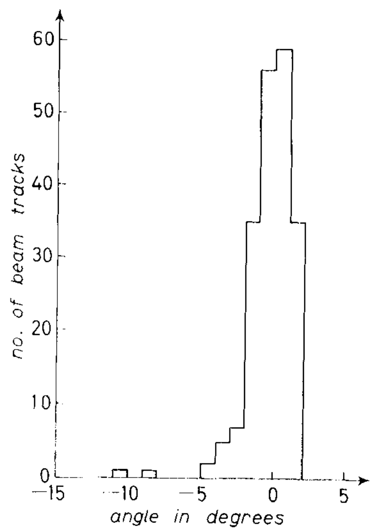

Fig. 2. – Angular distribution of muons in beam.

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The emulsions were processed at the Brookhaven facilities using standard techniques. A 1 mm grid was printed on the bottom of each pellicle to facilitate the recording and relocation of events.

The emulsions were scanned with standard binocular microscopes equipped with $20\times$ oil immersion objectives and $10\times$ or $15\times$ oculars. All stars were recorded and any star with a beam track passing through the center was listed by the scanner as a possible muon interaction. Only events with at least one dark or grey track were found by this method.

The major scanning effort was applied to the 5 GeV/c exposure where 8.4 cm³ of the perpendicular exposure and 2.7 cm³ of the parallel exposure were scanned. The effective muon track length examined was $9.5\cdot10^6$ cm. Of the plates exposed to 2.5 GeV/c muons, 3.3 cm³ of the perpendicular plates were scanned or an effective track length of $3.27\cdot10^6$ cm.

With very high track density there is nonnegligible probability for a beam track to coincide with a background star. A study of tracks from radioactive thorium decay in which beam tracks were coincident with stopping alpha-particles resulted in an estimated probability of 0.01 for a beam track to be coincident with a background star.

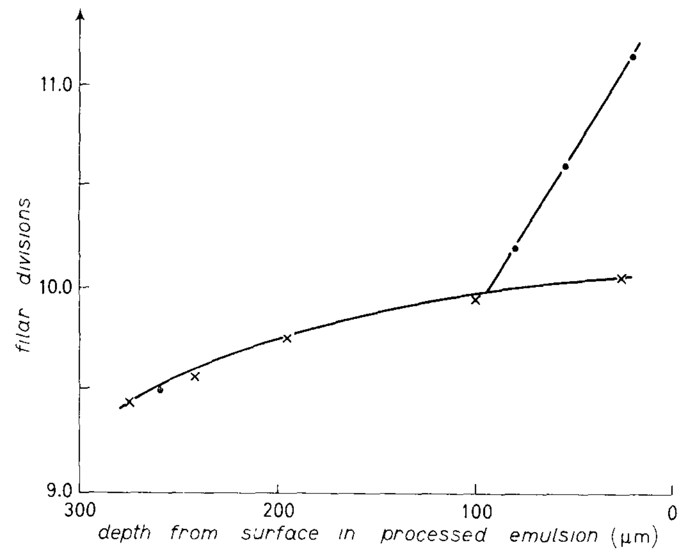

Fig. 3. – Measurement of angles for muon scattering event in emulsion: • event beam track; × nearby beam track.

Measurements of the azimuthal and dip angles of all secondary tracks were made on a Koristka measuring microscope. To obtain the scattering angle of the beam track on the plates with the perpendicular exposure, x and y

displacements were measured as a function of depth using a filar micrometer. Since the shrinkage of the emulsion results in a curvature of the beam tracks, measurements were made on a nearby beam as well as the event beam. By assuming the nearby beam to be undeviated, scatterings of the even beam could be observed with an accuracy of 0.3°. The results of a typical measurement is shown in Fig. 3. Grain density measurements were made on all grey and light secondary tracks. Corrections were made for dip angle and variation of grain density with depth in the emulsion. The grain densities of minimum tracks were measured and the ratio $g/g_{\min}$ computed. Multiple scattering measurements were made on all grey and light tracks using the method of Fowler. The measurements were made on a scattering stage constructed at Berkeley which is built on a set of precision ways to which the track is rotated into alignment and translated in a straight line. Measurements were made of the multiple Coulomb scattering using a filar micrometer which could be read to an accuracy of 0.02 μm. The identity of the track is made using the Sternheimer energy loss relation and the measurements of relative grain density and momentum.

3. – Results.

All stars with at least one grey or black track were recorded. If a muon beam track passed through the center of the star, the star was listed as a possible muon event. Table I lists all such events with two or more secondary particles. The high probability of a beam track being in coincidence with a background star, mentioned previously, suggests that there would be a considerable percentage of false events with a scattering angle $< 0.3°$ (the precision with which scattering angles can be determined). Approximately 2000 background stars were

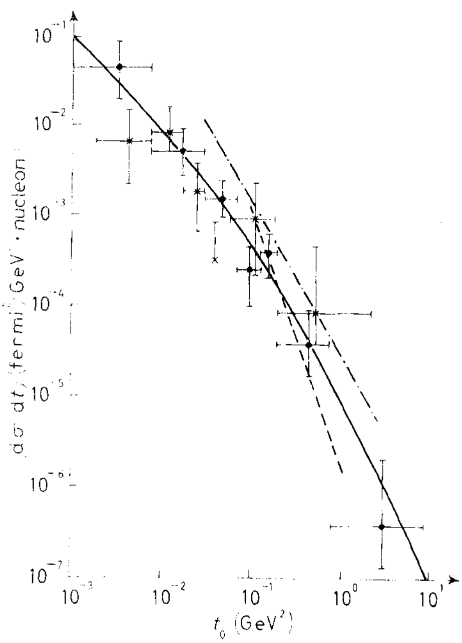

Fig. 4. – Experimental and theoretical inelastic scattering cross-sections for muons: × 2.5 GeV/c; • 5.0 GeV/c; —— inelastic scattering by transverse photons; —·—·— elastic scattering from point nucleons; – – – elastic scattering from extended nucleons.

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TABLE I. – *Muon interactions. Events with two or more prongs.*

Event No.	Muon defl. angle degrees	Type of interaction (*d*) $n_b + n_g + n_s$	Event No.	Muon defl. angle degrees	Type of interaction (*d*) $n_b + n_g + n_s$	Event No.	Muon defl. angle degrees	Type of interaction (*d*) $n_b + n_g + n_s$
1	< 0.3	$3+2+2$	41	< 0.3	$2+0+0$	81	0.8	$3+1+1$
2	< 0.3	$2+1+1$	42	0.6	$2+0+0$	82	< 0.3	$3+1+0$
3	> 10.0	$16+3+2$	43	1.1	$1+1+0$	83	0.4	$5+0+0$
4	0.6	$3+1+0$	44	1.3	$3+0+0$	84	1.9	$4+1+0$
5	(*a*)	$3+0+0$	45	(*a*)		85	< 0.3	$1+1+0$
6	0.9	$3+1+1$	46	1.2	$3+1+2$	86	< 0.3	$3+0+0$
7	0.3	$1+1+1$	47	< 0.3	$2+0+0$	87	3.9	$4+1+0$
8	(*a*)	$1+2+0$	48	0.8	$6+3+2$	88	2.7	$7+1+1$
9	0.3	$3+1+1$	49	2.5	$4+1+0$	89	7	$1+1+0$
10	< 0.3	$2+1+0$	50	1.1	$1+1+0$	90	< 0.3	$2+1+0$
11	1.3	$7+2+0$	51	< 0.3	$3+1+0$	91	0.6	$2+0+0$
12	2.2	$2+0+1$	52	< 0.3	$2+1+0$	92	< 0.3	$2+1+0$
13	(*a*)	$2+0+0$	53	< 0.3	$3+0+0$	93	0.3	$5+1+0$
14	5.5	$2+1+0$	54	0.9	$4+0+0$	94	(*)	$7+0+0$
15	< 0.3	$2+0+1$	55	0.4	$1+1+0$	95	0.9	$2+0+0$
16	4.2	$3+0+0$	56	1.8	$3+0+1$	96	3.2	$2+0+0$
17	< 0.3	$2+1+0$	57	< 0.3	$6+1+0$	97	0.8	$2+0+0$
18	< 0.3	$3+1+0$	58	0.4	$10+1+0$	98	< 0.3	$3+0+0$
19	2.8	$5+1+0$	59	0.5	$3+1+0$	99	(*)	$2+1+0$
20	0.8	$5+3+0$	60	(*c*)	$2+1+0$	100	< 0.3	$2+3+0$
21	1.1	$2+1+0$	61	< 0.3	$2+0+0$	101	(*)	$3+1+0$
22	> 10.0	$2+1+0$	62	0.4	$3+0+0$	102	0.8	$1+1+0$
23	< 0.3	$2+0+0$	63	< 0.3	$7+0+0$	103	< 0.3	$3+1+0$
24	0.6	$2+1+0$	64	(*)	$2+0+0$	104	0.8	$1+1+0$
25	< 0.3	$3+2+0$	65	(*)	$5+0+0$	105	< 0.3	$7+1+0$
26	< 0.3	$1+0+1$	66	1.2	$5+0+0$	106	0.3	$1+1+0$
27	4.4	$2+2+1$	67	< 0.3	$3+1+0$	107	0.9	$4+0+0$
28	(*a*)	$4+0+0$	68	0.8	$5+0+0$	108	4	$6+0+0$
29	1.0	$6+2+0$	69	> 10	$9+4+2$	109	0.8	$2+1+0$
30	< 0.3	$1+1+0$	70	3.4	$4+0+0$	110	< 0.3	$4+2+0$
31	1.2	$4+1+0$	71	0.4	$7+1+0$	112	< 0.3	$2+0+0$
32	0.4	$1+2+0$	72	< 0.3	$3+0+1$	111	< 0.3	$2+0+0$
33	< 0.3	$2+0+0$	73	4.3	$2+0+1$	113	< 0.3	$4+0+0$
34	1.0	$2+1+0$	74	2.8	$7+1+0$	114	< 0.3	$2+0+0$
35	6.8	$5+2+4$	75	< 0.3	$2+1+0$	115	> 10	$13+4+1$
36	7.1	$2+1+1$	76	4.2	$3+0+0$	116	0.4	$4+0+0$
37	0.3	$1+1+0$	77	(*b*)	$6+1+1$	117	0.3	$4+0+2$
38	< 0.3	$1+1+0$	78	2.6	$3+1+0$	118	0.6	$2+0+0$
39	< 0.3	$1+1+0$	79	1.3	$1+1+1$	119	1.4	$5+1+3$
40	9.1	$4+2+0$	80	< 0.3	$4+0+0$	120	4.8	$5+3+0$

(*a*) Near surface of emulsion.
(*b*) Two outgoing tracks.
(*c*) Two possible beam tracks.
(*d*) n_b = number of black tracks; n_g = number of grey tracks; n_s = number of light or shower tracks.

found. Since the estimated probability for a coincidence is 0.01, the expected number of false events with scattering angles less than 0.3° is 20. The numbers of dark, grey and light tracks are listed for each event. Light tracks were defined as those having grain densities of less than 1.5 times minimum. All tracks identified as pions were listed as light tracks.

TABLE II. – *Angular distribution of scattered muons.*

Angular interval	0.3° ÷ 1°	1° ÷ 2°	2° ÷ 3°	3° ÷ 4°	4° ÷ 5°	5° ÷ 10°	10° ÷ 180°
1-prong stars	25	12	7	0	0	0	0
stars with 2 or more prongs	31	14	6	3	6	5	4
Total	56	26	13	3	6	5	4

The angular distribution of all scattered muons is given in Table II. It is to be noted that the one-prong events which involve small energy transfers are limited to scattering angles of less than three degrees. The differential cross-section for inelastic muon scattering is presented as a function of four-momentum transfer in Fig. 4. It is to be noted that the experimental points on Fig. 4 represent interactions due to both 2.5 and 5.0 GeV/c muons. Within statistical errors, the inelastic muon cross-section in this experiment appears to be independent of the incoming energy and to depend only on the four-momentum transfer. The indicated errors include statistical errors as well as the error associated with the estimate of the four-momentum transfer.

The angular and energy distributions of the single secondary proton events are shown in Fig. 5 and 6. The angular distribution shows a maximum around

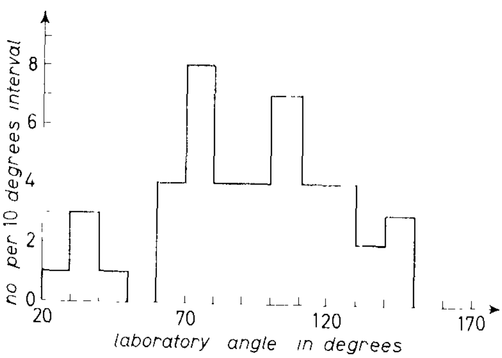

Fig. 5. – Energy spectrum of one-prong events (total of 26 events).

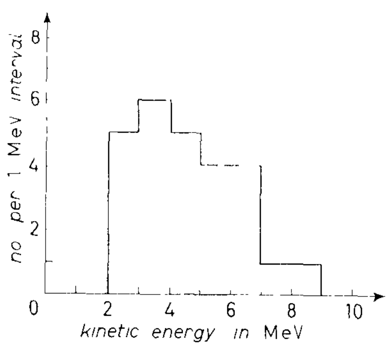

Fig. 6. – Angular distribution of one-prong events (total of 45 events).

90° while the energy is peaked around 5 MeV, similar to observations of the « giant » photodisintegration resonance ([25,26]). In Fig. 7, the momentum distribution of protons with momenta greater than 200 MeV/c shows no significant

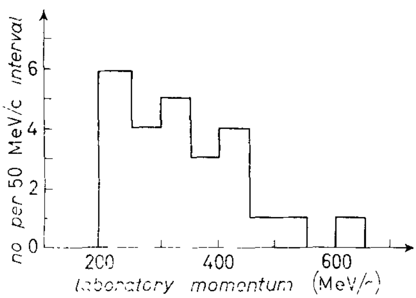

Fig. 7. – Momentum distribution of protons with momentum greater than 200 MeV/c (total of 25 events).

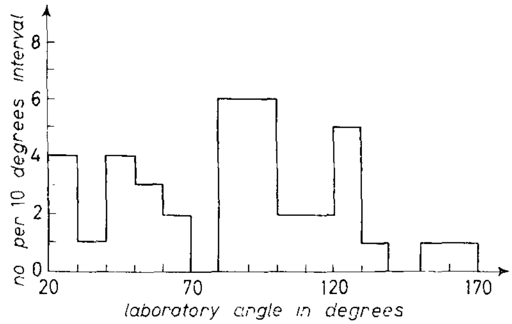

Fig. 8. – Angular distribution of single protons (total of 38 events).

peaking at higher momentum values. The angular distribution of all grey and light proton secondaries in Fig. 8 indicates a broad maximum at 90°.

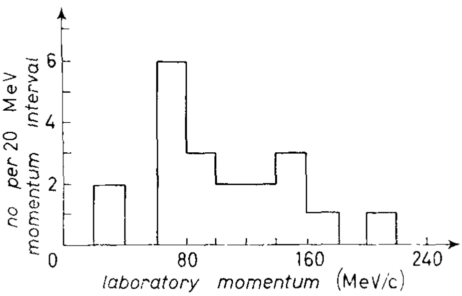

Fig. 9. – Momentum distribution of emitted pions (total of 20 events).

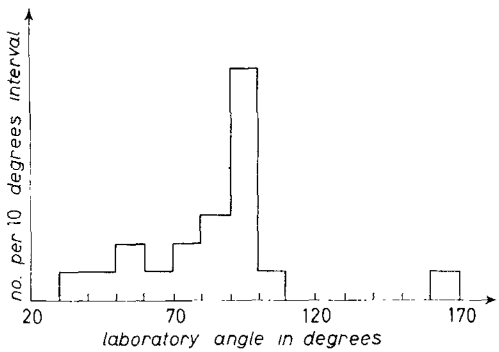

Fig. 10. – Angular distribution of emitted pions (total of 19 events).

The momentum and angular distribution of all identified secondary pions is shown in Fig. 9 and 10. The peak of the pion momentum distribution is found

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([25]) E. HAYWARD: *Rev. Mod. Phys.*, **35**, 324 (1963).
([26]) L. COHEN, A. K. MANN, B. J. PATTON, K. REIBEL, W. E. STEPHENS and E. J. WINHOLD: *Phys. Rev.*, **104**, 108 (1956).

at about 100 MeV/c in contrast to 240 MeV/c found for photoproduction from protons. However the pion momentum peak has been found to occur at lower momentum values for heavier nuclei ([27]). The general feature of the angular distributions of emitted particles is that the secondaries are preferably emitted at right angles to the muon beam.

4. – Discussion and conclusions.

The success of describing elastic electron-nucleon or muon-nucleon scattering in terms of two form factors has led to the speculation that a similar procedure should be useful in inelastic scattering. The Feynman diagram for the one-photon exchange is shown in Fig. 11*a*. A close correspondence is expected between this process involving virtual photons and the corresponding process involving real photons, Fig. 11*b*. The cross-section is factored into a kinematical part, with properties of gauge invariance and the vector nature of the photon, and a dynamical part which is due to the nucleon vertex and includes all of the complicated inelastic processes of disintegration and particle production. It is felt that quantum electrodynamics can be tested to high values of momentum transfer by observing inelastic scattering. The theory has been discussed in detail by HAND ([28]) and in a slightly different form by DRELL ([29]). The parameters used in this theory are defined in Fig. 12. In the figure $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the momenta of the incident and scattered muon. The momentum of the final state nucleon is $\boldsymbol{p}'$. E and E' are the corresponding total energies of the incident and scattered muon.

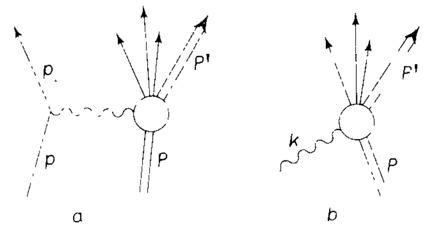

Fig. 11. – Angular diagrams for muon scattering process.

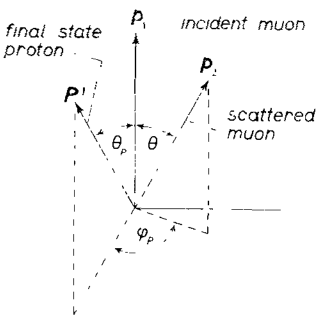

Fig. 12. – Angles used in analysis of data.

([27]) E. P. GEORGE: *Proc. Phys. Soc.*, A **69**, 110 (1956).
([28]) L. HAND and R. WILSON: Stanford Linear Accelerator Report 25 (1963).
([29]) S. D. DRELL and J. D. WALECKA: *Ann. Phys.*, **28**, 18 (1964).

The cross-section will involve the quantities:

(4.1) energy transfer $q_0 = E - E'$,

(4.2) momentum transfer $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$,

(4.3) four-momentum transfer $q = (q_0, \boldsymbol{q})$.

The square of the four-momentum transfer t is defined in terms of expression (A.5) in Appendix A:

$$t \equiv q^2 \simeq 2EE'(1-\cos\theta) + m^2(\cos\theta(E/E' + E'/E) - 2). \tag{4.4}$$

The angle θ as shown in Fig. 12 is the scattering angle of the lepton while m is the mass of the lepton. In this experiment m^2 is small compared to EE' so that the second term in (4.4) can be neglected. In addition, taking E' from expression (4.1) it follows that

$$t \simeq 2E^2\left(1 - \frac{q_0}{E}\right)(1-\cos\theta)\,. \tag{4.5}$$

Finally, in this experiment, $q_0 \ll E$ and θ is sufficiently small so that

$$t \simeq E^2\theta^2 \equiv t_0\,. \tag{4.6}$$

The approximate four-momentum transfer, expression (4.6), is quite accurate for small angles but results in an overestimate at larger ones. For scattering angles less than 10° and energy transfers less than 0.5 GeV, the inaccuracy of this approximation is less than 12 %. However, in an extreme case for an energy transfer of 2 GeV, the square of the four-momentum transfer, t, could be overestimated by 40 %. In calculating the experimental cross-sections shown in Fig. 4, the over-estimate of t_0 is likely to be of importance only for the last two points. For example, the last point in Fig. 4 has as its upper limit in t_0, the maximum value permitted for a muon-nucleon collision. An over-estimate of t_0 would tend to move this point to the right but produces little effect upon agreement with the solid curve.

As shown in Appendices B and C the inelastic differential cross-section assuming single-photon exchange in the region of small momentum and energy transfers, $q < q_0 < E$, can be written in terms of t_0:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}t_0\,\mathrm{d}q_0} = \frac{\alpha}{\pi}\frac{1}{q_0 t_0}\,\sigma_\gamma(q_0)\,, \qquad m^2 < q^2 < q_0^2, \tag{4.7}$$

where $\sigma_\gamma(q_0)$ is the corresponding cross-section observed for real photons.

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In the other case, where $q > q_0$ the cross-section from expression (B.7) for transverse virtual photons is given by

$$\frac{d^2\sigma}{dt\,dq_0} = \frac{\alpha}{\pi}\,\frac{K\sigma(q^2, K)}{t^2}\,. \tag{4.8}$$

However, it is not clear just what the effect of the scalar or longitudinal photons should be. This is an effect which must be determined experimentally. The only approach presently available is to attempt to account for the known existing cross-section due to transverse photons and to attribute additional effects to the longitudinal part.

The four-momentum transfer was determined with expression 4.6 for each of the muon interactions listed in Table II. The results were plotted in Fig. 4 where the cross-section is shown as a function of t_0. The solid curve through the points represents the theoretical results obtained from expression (B.7) in Appendix B. It was necessary to make a number of important assumptions in order to carry out the calculation. First, as indicated before, only transverse photon interactions were considered and the cross-section σ_γ in (B.7) was taken to be that for real photons. The general energy dependence of σ_γ was obtained from the work of WALKER (30) while the peak values for single pion production were taken to be

$$\gamma + p \to \pi^0 + p \qquad 270 \text{ microbarn},$$

$$\gamma + n \to \pi^0 + n \qquad 270 \text{ microbarn},$$

$$\gamma + p \to \pi^0 + n \qquad 230 \text{ microbarn},$$

$$h + n \to \pi^0 + p \qquad 300 \text{ microbarn}.$$

The cross-sections and their energy dependence are in good agreement with the experiments of CASTAGNOLI *et al.* (31) who studied the interactions produced by 1100 MeV bremsstrahlung radiation in emulsions. In addition, the assumption that emulsion nuclei can be treated as free protons and neutrons was found to be quite good. The contributions due to multiple pion production were unimportant in the energy range of this experiment and the energy exchange of the muon was assumed to be given by the energy of the virtual photon. Finally, with the above assumptions a numerical integration of eq. (B.7) was performed to obtain the solid curve in Fig. 4.

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(30) R. L. WALKER: *Proceedings of the Conference on Photon Interactions in the GeV-Energy Range*, Cambridge, Mass., (Jan. 26-30, 1963).

(31) C. CASTAGNOLI, M. MUCHNIK, G. GHIGO and R. RINZIVILLO: *Nuovo Cimento*, **16**, 683 (1960).

The dashed curve in Fig. 4 was added in order to make a comparison with elastic muon-nucleon scattering. The curve was obtained from the Rosenbluth formula with the nucleon form factors given by HERMAN and HOFSTADTER [32]. As seen in the figure, the inelastic scattering becomes comparable to the elastic for four-momentum transfers in excess of 600 MeV.

The proton energy spectrum of the one-prong events shown in Fig. 5 is similar to that observed for giant resonance interactions and very different from the spectrum for the two or more prong stars shown in Fig. 7. It is also observed that while the total cross-section for star production by muons as listed in Table III is greater at 5 GeV/c than at 2.5 GeV/c, the total cross-section

TABLE III. – *Muon cross-sections in emulsion.*

Muons	Volume of emulsion scanned cm^3	Muon flux tracks per cm^2	Cross-sections per nucleon in microbarn (units of 10^{-30} cm^2)			
			« meson production » events		« giant resonance » events	stars with three or more prongs
			experi-mental	theo-retical		
2.5 GeV/c	3.3	$1.0\cdot10^6$	3.6 ± 0.7	2.9	19.0	2.2 ± 0.6
5.0 GeV/c	11.1	$8.6\cdot10^6$	5.1 ± 0.5	4.3	21.0	3.6 ± 0.4
Underground cosmic-ray muons (20)						4.6 ± 0.05

for one-prong events does not change with incident muon energy. This was also observed in the gamma-ray experiments of CASTAGNOLI [31] and indicates a low-energy nuclear disintegration process. Therefore it is felt that the majority of the one-prong events are giant-resonance events and are due to virtual photon energies of less than 40 MeV. It is to be observed in Table II that the muon scatterings associated with the one-prong stars are small and confined to less than three degrees. This indicates a differential cross-section decreasing rapidly with four-momentum-transfer and is consistent with eq. (4.8).

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[32] R. HERMAN and R. HOFSTADTER: *High-Energy Electron Scattering Tables* (Stanford, 1960).

The cross-sections for muon interactions were obtained with the aid of the experimental data on prong distributions for stars given in Fig. 13. As discussed before, nearly all of the one-prong stars were due to virtual photons of energy in the vicinity of the « giant resonance » (15 to 25 MeV). The great majority of such interactions involve the emission of neutrons alone, and could not be found in this experiment. On the other hand, « meson production » results from interactions by virtual photons of energy over 150 MeV. It is to be remembered in this experiment that in many of the « meson production » events, the meson is reabsorbed in the emulsion nucleus. For instance, charged pions were emitted in only 17 % of these events. Since the above two types of interaction are rather distinct, their partial cross-sections have been determined separately.

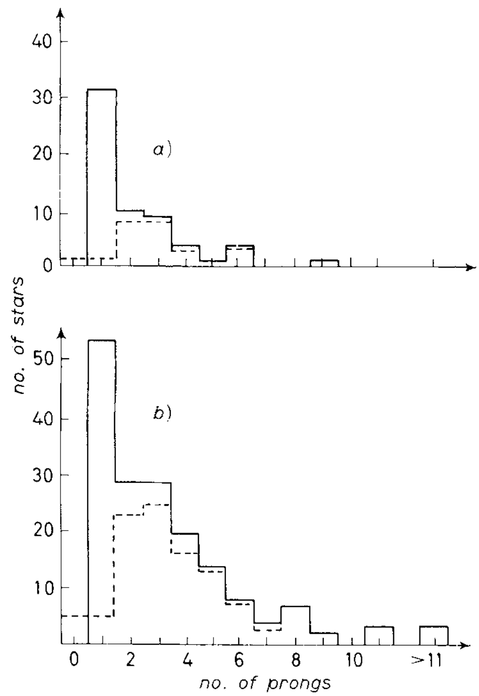

Fig. 13. – Prong distributions of *a*) 2.5 GeV/c and *b*) 5.0 GeV/c muon stars. ——— Observed histogram; – – – corrected histogram.

The solid curves shown in Fig. 13 represent the prong distribution for all muon interactions found in this experiment. The number of prongs does not include the incident or scattered muon so that direct comparison can be made with observations of real photonuclear stars. First, corrections of the data had to be made for scanning efficiency and, as described before, the probability that a muon beam track would pass through the center of a cosmic ray or background star. These corrections were small and are shown for stars with $\geqslant 2$ by the dashed histogram in Fig. 13. In order to separate the « giant resonance » and « meson production » stars, methods were employed which were similar to those used by GEORGE ([27]), PETERSON and ROOS ([33]), and CASTAGNOLI *et al.* ([31]) for photonuclear stars produced by bremsstrahlung radiation. In their experiments carried out from 300 to 1100 MeV, it was found that about 90 % of the 1-prong stars were associated with « giant resonance » reactions and, within statistical error, there were none with two or more prongs.

([33]) V. Z. PETERSON and C. E. ROOS: *Phys. Rev.*, **105**, 1620 (1957).

Since the muon virtual photon spectrum is quite similar to a bremsstrahlung spectrum, the same separation methods were used in the present experiments. The weighted average separation factors obtained from the above three bremsstrahlung experiments calls for 18% of the observed one-prong events to represent the sum of zero- and one-prong stars to be associated with «meson production» events. The error associated with this factor is hard to estimate but its effect upon the cross-section for «meson production» events is somewhat less than that due to the statistics for this experiment. Finally, the complete dashed histograms in Fig. 13, with all of the above corrections, represent the prong distribution for events of the «meson production» type.

The cross-sections for «meson production» events obtained from the dashed histograms in Fig. 13 are given in column 4 of Table III. The theoretical values given in column 5 of the Table were obtained by a numerical integration with the muon virtual photon spectrum of Kessler and Kessler [34] and the real photon cross-sections described above. While there is good agreement with the energy dependence of the cross-sections, the theoretical values are about 15% too low.

The cross-sections for «giant resonance» events given in column 6 of Table III can only be considered as crude estimates. The number of one-prong events associated with the «giant resonance» was directly obtained as indicated above. Then an estimate had to be made for the number of zero-prong events. For this purpose the photonuclear cross-sections of emulsion nuclei had to be obtained. While this information was available for some of the nuclei, rather uncertain estimates and extrapolations had to be made for others. A numerical integration (was then made of the cross-sections times photon spectrum over the «giant resonance» region and it was estimated that the ratio of zero-prong stars to one-prong stars was 5.2. This approximate ratio then made it possible to obtain the cross-sections given in column 6 of Table III.

It is possible to compare the cross-sections determined in this experiment with similar cosmic-ray observations. For instance, GEORGE *et al.* [20], as described in Sect. 1 of this paper, examined stars of 3 or more prongs produced by cosmic-ray muons underground. The average energy in their experiment was about 2 GeV and the corresponding cross-section was determined to be $(4.6 \pm 0.05) \cdot 10^{-30}\ \text{cm}^2$. This observation, as shown in Table III, is in reasonable agreement with the cross-sections determined in the present experiment. It is to be noted that the moderately strong energy dependence of the cross-section shown in column 7 indicates that the high-energy tail of the cosmic-ray muons will tend to make their average cross-section even higher than the present observations at 5 GeV/c.

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[34] D. KESSLER and P. KESSLER: *Nuovo Cimento*, 4, 601 (1956).

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APPENDIX A

The four momentum transfer is defined to be

((A.1) $$q^2 = (\Delta \boldsymbol{P})^2 - (\Delta E)^2 .$$

The cosine law can be used to relate the change in momentum to the scattering angle:

(A.2) $$(\Delta \boldsymbol{P})^2 = |\boldsymbol{P}|^2 + |\boldsymbol{P}'|^2 - 2|\boldsymbol{P}||\boldsymbol{P}'|\cos\theta ,$$

(A.3) $$q^2 = 2EE' - 2|\boldsymbol{P}||\boldsymbol{P}'|\cos\theta - 2m^2 ,$$

(A.4) $$|\boldsymbol{P}| = E^2 - m^2 \approx E - \tfrac{1}{2}m^2/E , \qquad \text{for } E \gg m .$$

(A.5) $$q^2 \approx 2EE'(1 - \cos\theta) + m^2\big(\cos\theta(E/E' + E'/E) - 2\big) .$$

The minimum value of q^2 occurs for $\theta = 0$:

(A.6) $$q^2_{\text{min.}} = m^2 q_0^2/EE' ,$$

where $q_0 = E - E'$.

APPENDIX B

HAND ([28]) expresses the inelastic differential cross-section in terms of the real photon cross-section:

(B.1) $$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \Gamma_{\text{transverse}}(\theta, q^2, K)\sigma_{\text{transverse}}(q^2, K) + \Gamma_{\text{scalar}}(\theta, q^2, K)\sigma_{\text{scalar}}(q^2, K) ,$$

where

$$\Gamma_{\text{transverse}} = \frac{KE'}{4\pi^2 q^2 E}\left[2 + \frac{\operatorname{ctg}\theta/2}{1 + q_0^2/q^2}\right], \tag{B.2}$$

$$\Gamma_{\text{scalar}} = \frac{KE'}{4\pi^2 q^2 E}\,\frac{\operatorname{ctg}^2\theta/2}{1 + q_0^2/q^2}, \tag{B.3}$$

where K is expressed by

$$K = E - E' - q^2/2M. \tag{B.4}$$

$\sigma_{\text{transverse}}(q^2 = 0)$ is the cross-section measured with real photons. σ_{scalar}, the scalar photoproduction cross-section, cannot be measured with real photons. The Γ's have the dimensions: number of virtual photons/MeV·sr.

From expression (A.5):

$$t = q^2 = 2EE'(1 - \cos\theta), \tag{B.5}$$

$$\mathrm{d}t = 2EE'\sin\theta\,\mathrm{d}\theta. \tag{B.6}$$

For small-angle scattering, the second term in the brackets of expression (B.2) dominates, and the cross-section becomes

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}q_0} = \frac{\alpha}{\pi}\frac{E'K\sigma_t(q^2, K)}{Et(t+q_0^2)} + \frac{\alpha}{\pi}\frac{E'\,K\sigma_s(q^2,K)}{Et(t+q_0^2)}. \tag{B.7}$$

Appendix C

Drell writes the differential inelastic cross-section as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E_f} = \frac{4Z^2\alpha^2E'^2\cos^2\theta/2}{q^4M_t}\left[W_2(q^2, q\cdot P) + 2W(q^2, q\cdot P)\operatorname{tg}^2\theta/2\right], \tag{C.1}$$

where

$$q\cdot P/M_t = E - E' = q_0. \tag{C.2}$$

Relating the matrix elements for the scalar and transverse terms, the form factors are expressed as

$$W_1(q^2, q\cdot P) = \frac{(q\cdot P)^2}{(2\pi)^2Z^2}\,\sigma(q\cdot P/M_t) + O(q^2), \tag{C.3}$$

$$W_2(q^2, q\cdot P) = \frac{M_t^2q^2}{(q\cdot P)^2}\,\frac{\sigma(q\cdot P/M_t)}{(2\pi)^2Z^2} + O(q^4). \tag{C.4}$$

Using the relations

$$\text{(C.5)} \qquad \begin{cases} d\Omega' = \tau\, dt/EE' & \text{for } q > m, \\ \cos^2\theta/2 = 1 & \text{for small angles.} \end{cases}$$

The cross-section can be expressed in the form

$$\text{(C.6)} \qquad \frac{d^2\sigma}{dt\,dE_f} = \frac{\alpha}{\pi}\frac{1}{t}\frac{E'}{E}\frac{\sigma_\gamma(q_0)}{q_0}\left[1 + \frac{q_0^0}{2EE'}\right].$$

RIASSUNTO (*)

Si è misurato lo scattering anelastico di muoni di alta energia sui nuclei dell'emulsione facendo uso di un fascio monoenergetico di muoni di 2.5 GeV/c e 5.0 GeV/c prodotto nel sincrotrone a gradiente alternato di Brookhaven. Si sono esposti pacchi di Ilford G-5 perpendicolarmente al fascio e pacchi di Ilford K-5 parallelamente al fascio. L'esplorazione di superficie diede un totale di 135 eventi con quadrimomento trasferito maggiore di 26 MeV/c. La distribuzione di scattering osservata che si estende sino a 10 gradi, o ad un momento trasferito di circa 900 MeV/c, è in buon accordo con le predizioni della teoria elettromagnetica nella ipotesi di scambio di un solo fotone. Furono osservati anche quattro eventi con angoli di scattering maggiori di 10 gradi che non discordano con la teoria. Si è riscontrata produzione di pioni cui il contributo maggiore derivava dalla risonanza $\frac{3}{2}, \frac{3}{2}$. Si è trovato che le sezioni d'urto per questo processo sono (3.6 ± 0.7) e (5.1 ± 0.5) microbarn per muoni di 2.5 e 5.0 GeV/c, rispettivamente. Si è trovato che gli eventi che comportano scambi di energia molto minori hanno sezioni d'urto molto maggiori e concordano con le osservazioni della fotodisintegrazione della « risonanza gigante ».

(*) *Traduzione a cura della Redazione.*