Multiparticle Production of Doubly Charged Fragments in the Fragmentation of Relativistic Nuclei

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Abstract—Features of angular distributions of events involving two or more doubly charged fragments of relativistic nuclei \(^{22}\text{Ne},^{24}\text{Mg},^{14}\text{N},^{11}\text{B},^{10}\text{B}\) in photoemulsions are studied. It is found that, in all cases, with the exception of the case of the intermediate-state decay \(^{8}\text{Be} \rightarrow 2\alpha\), the fragments in these events are independent of one another. The inclusive angular distributions of fragments of relativistic nuclei \(^{22}\text{Ne}\) for events in which the number of particles ranges between one and five are identical. Thus, the emission angle of each fragment of a relativistic nucleus does not depend either on other fragments or on the presence or absence of product particles and target-nucleus fragments in an event.

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I. INTRODUCTION

At the present time, there is a vast body of experimental data on the fragmentation of relativistic nuclei ranging between lithium and lead, having energies between 1.2 and 200 GeV, and interacting with photoemulsion nuclei [1–8]. Doubly charged fragments stand out among all possible fragments. The cross sections for their production are large, and their identification is highly reliable. While singly charged particles appearing in a narrow cone of secondaries may involve not only ions but also product pions, all traces featuring quadruple ionization are exclusively helium isotopes. As was shown in [3], the isotope \(^{6}\text{He}\) can also be separated in the fragmentation of relativistic nuclei \(^{11}\text{B}\). The channel \(^{10}\text{B} \rightarrow ^{8}\text{Be} + \text{all} \rightarrow 2\alpha + \text{all}\) was selected in [2] in the fragmentation of \(^{10}\text{B}\) nuclei, and the fact of a cascade fragmentation of a relativistic nucleus was established in this way. In [8], a \(^{12}\text{C}\) intermediate state decaying to three alpha particles was assumed to contribute to the fragmentation channel \(^{14}\text{N} \rightarrow 3\alpha + \text{all}\), but no piece of quantitative of evidence was obtained for this there.

The problem of the existence of resonance states formed by three or more alpha particles is of interest. In view of this, almost the same events as those that were found in the Laboratory of High Energies at the Joint Institute for Nuclear Research (JINR Dubna) in scanning along a track were revisited in the present study by applying the same procedure for measuring angles as that which was employed in studying the fragmentation of relativistic nuclei in [2, 3, 5]. The results obtained in this way are discussed below.

In the present article, particular attention is given to experimental data on the fragmentation of relativistic nuclei \(^{22}\text{Ne}\) having a momentum of 4.1 GeV/c per nucleon and interacting with photoemulsion nuclei [1]. The database of 4309 respective events can be found in [9]. The fragmentation of relativistic neon nuclei was only a small part of the research program of the collaboration that composed this database. An analysis of data in 16 emulsion chambers revealed that, in some them, there are events in which emission angles of doubly charged fragments differ from the average angles for the bulk of such fragments by an order of magnitude. All of them were excluded from the set events [10], and the data sample obtained in this way was used in the present study.

In an emulsion chamber exposed to \(^{24}\text{Mg}\) nuclei of momentum 4.1 GeV/c per nucleon, 65 events featuring four alpha particles each were chosen among events found by viewing along a track. For all of them, emission angles in each event were measured according to the same procedure as that which was used in [2, 3, 5]. There is a free access to the array of these data [11].

Thus, all of the data subjected to the analysis here, with the exception of the data on the fragmentation of \(^{22}\text{Ne}\) nuclei, were obtained by the same experimentalist according to the same measurement procedure. Of course, all primary data on emission angles of doubly charged fragments of various relativistic nuclei contain information much richer than that which is
employed in the present study. This information is accessible and may be of use to all who are interested in the mechanism of fragmentation of relativistic nuclei.

2. EXPECTED RESULTS

In the fragmentation of a relativistic nucleus that have a mass number \( A_0 \), a priori information about angular features of fragments with a mass number \( A_f \) is that the variances \( \sigma_\varphi^2 \) and \( \sigma_\alpha^2 \) of angles of the distributions of transverse-momentum projections onto any two mutually orthogonal directions must be identical. Since the angles \( \varphi \) and \( \alpha \) are small in all our experiments, we will henceforth set

\[
\tan(\varphi, \alpha) \simeq \sin(\varphi, \alpha) \simeq \varphi, \alpha.
\]

In this case, the polar angle is given by \( \theta = \sqrt{\varphi^2 + \alpha^2} \), while the fragment transverse momentum \( p_{\perp, f} = A_f P_\varphi \theta_f \) is obtained by multiplying the directly measured angle by constants. According to [12], the variance of the projections of fragment transverse momenta onto an arbitrary direction in the reference frame comoving with the center of mass of two colliding nuclei has the form

\[
\sigma^2(p_{\perp, \varphi, \alpha}) = \frac{\sigma_0^2 A_f (A_0 - A_f)}{A_0 - 1},
\]

where \( \sigma_0^2 = P_F^2/5 \) is the variance of intranuclear nucleons whose Fermi momentum \( P_F \) is that for a relativistic nucleus of mass number \( A_0 \) prior to its interaction with a photoemulsion nucleus. The values of \( P_F \) for various nuclei were determined in experiments that studied electron–nucleus scattering [13]. It turns out that, in any experiment that deals with the fragmentation of relativistic nuclei, angular distributions of fragments can be predicted before performing this experiment without any free parameters. This has already been confirmed both in numerous photoemulsion experiments and in electron experiments (see, for example, [14]).

For a number of reasons, the photoemulsion procedure gives no way to measure, with the same precision, the angles \( \varphi \) and \( \alpha \) in the emulsion plane and a plane orthogonal to it. In view of this, the distribution of fragment transverse momenta frequently proves to be different from the expected \( \chi^2 \) distribution (Rayleigh distribution). There appear fragments having high transverse momenta. Of course, this is so only for a uniform data sample, in which case the distribution of the transverse-momentum projection onto an arbitrary direction is a normal distribution characterized by zero mean value. But if we deal with a mixture of two normal distributions from two sources of fragments (as in the experiment reported in [5]), then the resulting pattern is much more involved. Such an effect was first observed in the electron experiment of Bertullani and Hussein [15].

However, this law in the photoemulsion procedure, where only one angle \( \varphi \) in the emulsion plane is measured accurately, does not prevent a reliable evaluation of the mean transverse momentum of fragments even without performing measurements for each of them. It is clear that

\[
\langle p_{\perp, f} \rangle = \sqrt{\pi} A_0 p_\varphi \sigma_\varphi
\]

and that the mean kinetic energy in the reference frame comoving with the fragmenting nucleus is

\[
\langle T_f \rangle = (3/2)(\langle p_{\perp, f} \rangle^2/(2m_f)).
\]

All physical quantities of interest that characterize the set of fragments having a rest mass \( m_f \) can be obtained from the variance of the distribution of angles \( \varphi \) alone without measuring these quantities in each individual event.

Naturally, the distribution of azimuthal angles of fragments in the transverse plane,

\[
\Psi = \arctan \frac{\varphi}{\alpha},
\]

in the absence of polarization in our experiments must be uniform.

The distribution of the pair azimuthal angle

\[
\Delta \Psi_{i,j} = |\Psi_i - \Psi_j|
\]

between the transverse-momentum vectors may be either uniform in the case of independent particle divergence [16] or nonuniform because of trivial kinematical correlations [17]. In the latter case, there is an excess of pair angles in the region \( \Delta \Psi_{i,j} > 90^\circ \). Moreover, dynamical correlations in the fragmentation of relativistic carbon nuclei through the channel

\[ ^{12}C \to ^8\text{Be} + \text{all} \to 2\alpha + \text{all} \]

were also observed [18]. The calculated branching fraction of this channel is large [19].

The aforementioned dynamical correlations of the magnitudes and directions of transverse momenta are also observed in the reference frame comoving with the center of mass of the particles. Kinematical correlations, which are absent in the laboratory frame if particles stay apart independently, appear after going over to the reference frame where the vector sum of transverse momenta is zero. The respective transformation is usually made by the formula

\[
p^*_{\perp, i} = p_{\perp, i} - \frac{\sum p_{\perp, i, i}}{n},
\]

where the vector sum of \( n \) transverse momenta in an event appears in the numerator (see, for example, [7, 8]). As was indicated in [3], it is impossible in a
photoemulsion experiment to go over to the reference frame comoving with the center of mass of fragments. The respective transformation is merely a transition to the reference frame where the vector sum of transverse momenta is zero. In [20], a simulation revealed that, in the case being considered, we have

$$\eta = \frac{\langle p_\perp \rangle}{\langle p^*_\perp \rangle} = \sqrt{2}. \quad (5)$$

As was shown in [7], the experimental value of the ratio in (5) is indeed $\sqrt{2}$. It follows that the excess of particles that was observed in [8] for pair azimuthal angles smaller than 90 degrees is a mere corollary of the transformation in (4).

Let us now dwell briefly upon other predictions that were made for the divergence of independent particles in an event and which will be tested experimentally in what follows. In the absence of dynamical correlations, the variance of the sum of $n$ independent angles $\varphi$ is $n \sigma_\varphi^2$, so that the distribution of the angles $\theta_{i,j}$ between pairs of traces must be identical to the distribution of the angles $\theta$ themselves. On a logarithmic scale, the dependence of the probability of observing the square of a pair angle in excess of a preset value on its square must be linear. In [3], this was already confirmed for the fragmentation process $^{10}\text{B} \rightarrow 2\alpha + \text{all}$. Let us now proceed to consider different relativistic nuclei fragmenting into two or more doubly charged particles accompanied by an unknown number of other fragments, product particles, and products of target-nucleus dissociation.

### Table 1. Expected ($\sigma_\varphi$) and experimental ($\sigma_\varphi$ and $\sigma_\alpha$) constants of normal distributions of the angles $\varphi$ and $\alpha$ for doubly charged fragments of relativistic nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$p_0$</th>
<th>$\sigma_c$</th>
<th>$\sigma_\varphi$</th>
<th>$\sigma_\alpha$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}\text{B}$</td>
<td>1.7</td>
<td>21.0*</td>
<td>21.9 ± 0.6</td>
<td>20.1 ± 0.6</td>
<td>[2]</td>
</tr>
<tr>
<td>$^{11}\text{B}$</td>
<td>2.9</td>
<td>12.0**</td>
<td>12.5 ± 0.4</td>
<td>12.7 ± 0.4</td>
<td>[3]</td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>2.9</td>
<td>16.0***</td>
<td>10.6 ± 0.3</td>
<td>12.7 ± 0.3</td>
<td>[21]</td>
</tr>
<tr>
<td>$^{22}\text{Ne}$</td>
<td>4.1</td>
<td>11.8</td>
<td>11.6 ± 0.4</td>
<td>12.7 ± 0.4</td>
<td>[10]</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>200</td>
<td>0.2501</td>
<td>0.2601 ± 0.013</td>
<td>–</td>
<td>[4]</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>4.1</td>
<td>12.0</td>
<td>10.7 ± 0.7</td>
<td>13.1 ± 0.3</td>
<td>[11]</td>
</tr>
<tr>
<td>Pb</td>
<td>160</td>
<td>0.364</td>
<td>0.37 ± 0.02</td>
<td>–</td>
<td>[6]</td>
</tr>
</tbody>
</table>

* For particles not originating from the $^8\text{Be}$ intermediate channel.
** For particles identified as $^4\text{He}$.
*** Result of the present study for the channel $^{14}\text{N} \rightarrow 3\alpha + \text{all}$.

3. INCLUSIVE DISTRIBUTIONS OF ANGLES

Experimental values of the standard deviations of distributions of angles $\varphi$ and $\alpha$ for various relativistic nuclei are given in Table 1. Within the errors, we have $\sigma_\varphi \sim \sigma_\alpha$; moreover, all distributions comply well with a normal distribution, their variance being dependent on the Fermi momentum and atomic number of the relativistic nucleus being considered. Obviously, the distribution of fragment transverse momenta is obtained from the distribution of angles by multiplying its ordinate by a constant. All of these distributions can be obtained even before performing a relevant experiment. This is reason why it is the discrepancy between experimental results and theoretical predictions rather than their agreement that is of interest.

This is precisely what is observed in the experiments that study the fragmentation of $^{14}\text{N}$ nuclei, in which case experimental estimates of the constant in the normal distribution of angles for doubly charged fragments proved to be markedly smaller than the expected value.

No errors in measuring angles may lead to a decrease in the variance of the respective distribution. The presence of the isotopes $^3\text{He}$ and $^4\text{He}$ cannot reduce this dispersion sizably either. One of these isotopes decreases the standard deviation of the distribution, while the other increases it, but, in either case, the change is about 10%, so that it is impossible to notice this on the basis of the available statistical data sample.

It only remains to seek physical reasons behind the observed phenomenon. The assumption that a mixture of two normal distributions characterized by different variances is present in the experiment was verified in [22]. One of these variances is equal to the expected variance, while the other is a first free parameter that makes it possible to attain better agreement with experimental data. The weight of the normal distribution having this parameter value is a sec-
momentum is higher. All this was observed in the fragmentation of $^6$Li nuclei [5].

The distributions of angles $\varphi$ and $\alpha$ for fragments of a $^{22}$Ne relativistic nucleus in events involving one to five doubly charged fragments do not differ from each other and admit a good description by a normal distribution with a constant predicted for this nucleus, this indicating that the fragments are independent of each other. For this reason, we will consider in more detail the proof of the independence of fragment emission angles in an event.

4. ANGULAR CORRELATIONS

Almost all of the experimental studies devoted to the fragmentation of relativistic nuclei contain the statement that, in the reference frame comoving with the fragmenting nucleus, it decays from an excited state, but no proof of this statement does in fact exist. It is common practice to assume that the greater the number of secondary particles, including target-nucleus fragments and product particles, the higher the excitation energy. Therefore, one associates so-called wide stars [24], for example, which involve only relativistic-nucleus fragments, with low-excitation-energy events.

These concepts take origin in the Bohr–Frenkel era, when the prevalent opinion was that a compound nucleus emits observed nuclear-reaction products upon absorbing the projectile particle. Immediately after the discovery of the multiparticle-production phenomenon in the Fermi and Landau models, this pattern was extended to proton–proton interaction: the formation of a high-temperature cloud there was followed by pion emission from it in the cooling process. However, the experimental observation of the invariability of product–particle transverse momenta stopped a further development of the first models of the mechanism of multiparticle production via the decay of an excited state.

The present-day concept of the decay of an excited relativistic nucleus to fragments bears a close resemblance to the early multiparticle-production model. We have seen that inclusive distributions of transverse momenta of fragments of a relativistic nucleus are independent of its energy. It is also well known that, in any process, the vector sum of the transverse momenta of relativistic fragments in an event is not zero. Moreover, the analysis in [25] revealed that, in the fragmentation of $^{22}$Ne relativistic nuclei to two or more fragments, the modulus of the vector sum of $k$ transverse momenta, where $k$ is the number of fragments in such events, complies with the Rayleigh distribution characterized by the constant

$$
\Sigma = \sqrt{\sigma_1^2 + \cdots + \sigma_k^2}.
$$

As a consequence of an infinity
divisibility of a normal distribution, the variance of the sum is equal to the sum of variances in the case of independent emission.

For sets of events under study, which involve the emission of two or more doubly charged fragments, the expected standard deviations of the sums of angles $\varphi$ for the case of two to five fragments in an event are listed in Table 2 along with their experimental estimates, the errors in the latter also being indicated. One can see that no set of experimental data disproves the hypothesis of statistical independence of the fragment emission direction. There is even no hint of inevitable kinematical correlations at a small number of fragments. By no means does this of course mean the violation of the momentum-conservation law. Merely, fragments that we observe are only some part of the unknown total set of particles, for which the momentum-conservation law holds. The point is that we choose a small number of particles at random, and the vector sum of transverse momenta must not be zero for them. The momentum-conservation law holds only for the projectile and the target nucleus taken together rather than for each of them individually, as would be the case in the decay of two excited nuclei.

In all probability, fragmentation proceeds through a one-stage mechanism, and the hypothesis of the decay of an excited relativistic nucleus contradicts experimental data. The fragmentation process is fast and cold [26]. Limiting fragmentation similar to limiting fragmentation in hadron–hadron interactions [27] proceeds.

5. ANGLES BETWEEN PAIRS OF TRACKS

If the angles $\varphi$ and $\alpha$ in an event obey a normal distribution characterized by zero mean value and equal variances and are independent, the distribution of the spatial angle $\theta_{12}$ between all combinations of pairs of tracks in an event is identical to the inclusive distribution of the angles $\theta = \sqrt{\varphi^2 + \alpha^2}$. It is expected to be a Rayleigh distribution with a constant $\sigma_\varphi$. It does not carry any new information in relation to the distribution of the angle $\theta$.

However, the angle $\theta_{12}$ is of interest in that it determines, apart from a constant, the so-called excitation energy $Q$ introduced in [7, 8]. Indeed, this is the sum of the kinetic energies of the transverse motion of two alpha particles in our case; that is,

$$Q = \theta_{12}^2 \frac{4\gamma^2}{m_\alpha}. \tag{6}$$

Clearly, $Q$ defined in this way cannot be the excitation energy even if the channel threshold energy is added to it, as was done in [28].

Naturally, the distribution density for $Q$ (or $\theta_{12}$) must be a smooth function. It must have no maxima. As $Q$ decreases, the distribution density in question tends to zero, in just the same way as its counterpart for transverse momenta of fragments of relativistic nuclei does. If the histogram of experimental estimates of any of these two quantities does not have a dip at the origin, the reasons behind this are purely methodological, but they have nothing to do with new physics.

Experimental estimates of the angles $\theta$ and $\theta_{12}$ are always biased toward greater values. There are no unbiased estimates for them if the angles are measured by the coordinate method. Here, the smaller the angle, the greater the relative bias. The shape of the histograms for these quantities depends not only on the accuracy in measuring the angles $\varphi$ and $\alpha$ but also on the width of the histogram channel and on the volume of the data sample. If this volume is small and if the channel width is large, there is no hint of a Rayleigh distribution for this quantity.

However, the fragmentation of a relativistic nucleus may violate this pattern if the $^9$Be $\rightarrow$ 2$\alpha$ intermediate-
state branching fraction is large and if the accuracy in estimating the angle $\theta_{12}$ makes it possible to separate the intermediate state in question, as was done in [2]. In the fragmentation of $^{22}$Ne relativistic nuclei, the angle between the tracks of doubly charged particles is smaller than 2 mrad. In the experiment being considered, one can see, in the region of small pair angles between the tracks, only the background caused by the errors in measuring these angles. At the same time, the distribution of these angles in the $^{10,11}$B \( \rightarrow \) 2$(Z = 2) + \text{all relativistic-nucleus-fragmentation process in Fig. 2 is in good qualitative agreement with the prediction.}

6. AZIMUTHAL ANGLES

The fact that, in any photoemulsion experiment, the angles in two mutually orthogonal planes are always measured under different conditions inevitably affects the distribution of the transverse momenta of fragments and angles between them in the transverse plane.

For purely technical reasons, the distribution of the azimuthal angle $\Psi$ cannot be uniform in a photoemulsion experiment if the statistical data sample is quite wide. This was observed in the fragmentation of $^{22}$Ne relativistic nuclei, and the same can be seen in Fig. 3 for the fragmentation of $^{10,11}$B nuclei. The maxima at $90^\circ$ and $270^\circ$ imply that the average values of the angles are larger in the vertical than in the horizontal plane. If the statistical sample is small, then the distribution of the angles $\Psi$ may be indistinguishable from a uniform distribution. But if we require that $\Psi \rightarrow 0$ for $\alpha \rightarrow 0$, then peaks appear at angles of $\Psi = 0^\circ$ and $360^\circ$.

Obviously, all these circumstances also affect the distribution of the angles $\Delta\Psi_{i,j}$ between the transverse-momentum vectors. Moreover, the accuracy in experimentally determining the direction of the primary track has a nontrivial effect on this distribution. In [29], it was shown that, for any deviation from the true position of the point of intersection of the transverse plane with the primary track, the number of angles satisfying the condition $\Delta\Psi_{i,j} < 90^\circ$ is greater than that in the absence of this deviation even in the case of the emission of independent particles. It is obvious that, if $\sigma(\alpha_0) \sim \sigma(\alpha)$, these distortions cannot be disregarded. The distribution of angles also depends on the conditions under which the chamber is irradiated with relativistic ions.

Figure 4 displays the distributions of the angles $\Delta\Psi_{i,j}$ in the fragmentation of $^{10,11}$B relativistic nuclei. These distributions proved to be different: for $^{11}$B nuclei (histogram), there are a greater number of events in the left part of the figure, while, for $^{10}$B nuclei (points), there are a greater number of events in its right part. The latter may be due to the presence of kinematical correlations, while, in the fragmentation of $^{11}$B nuclei, dynamical correlations of the type of coalescence of two doubly charged fragments may be operative. But in all probability, this is caused by the distinction between the two chambers used, so that all special features are associated with the procedure for measuring angles for fragments in two mutually orthogonal planes. Independent proofs that the observed effects are real are required.

7. CONCLUSIONS

In the fragmentation of various relativistic nuclei having different energies, experimental data on angular distributions of particles in events featuring two or more doubly charged fragments do not comply
with the idea that fragments originate from the decay of an excited nucleus. There is no piece of evidence suggesting the existence of any "prefragments," with the exception of a $^8$Be nucleus, $^8$Be $\rightarrow 2\alpha$, that decay to two or more helium isotopes.

Of course, a thermodynamic formalism as a means for describing observables is possible and may be of use. However, this approach requires introducing free parameters not observed and not measured directly in experiments. A satisfactory description of experimental data on this basis after performing an experiment cannot serve as a proof that these free parameters do indeed exist. There is no proof of the uniqueness of this description. In view of this, the thermodynamic formalism cannot claim for the role of a theory of the fragmentation of relativistic nuclei.

In all probability, such a theory cannot be developed in isolation from the theory of multiparticle production and the ideas of fragmentation of quarks and hadrons at ultrahigh energies. Possibly, the only difference is that, in the fragmentation of relativistic nuclei, all processes proceed very close to the mass shell.

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REFERENCES


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