«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
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ON SYMMETRY IN MODERN PHYSICS (Dedicated to the 100th anniversary of the birth of Academician V.A.Fock)

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The development of the gauge symmetry has resulted in a complete determination of the Lagrangians for electromagnetic, weak, strong and gravitational interactions and has created illusions about the construction of «the theory of everything». However, in just the same way as in classical physics, it became clear that the deductive obtaining of solutions (laws of Nature) is based not only on the principles of the Lagrangian symmetry. To find unambiguously solutions some *additional conditions* are needed without which the solutions of the Lagrange equations are ambiguous. The additional conditions such as hypotheses about the integral symmetries of solutions, the boundary and initial conditions, the constants entering Lagrangians, and so on are essential so that in a number of cases it is possible to construct models (solutions, laws of Nature) without the recourse to the Lagrange method. An example of using such an approach in one of the rapidly developing domains of modern physics, namely relativistic nuclear physics, is given. An exact mathematical language of the gauge symmetry is the differential geometry and that of the additional conditions is the topology, the parameter space properties as a whole. In the present paper the fundamental contribution of V.A.Fock to the development of the concept of space, the primary concept of physics, is given.

Разработка калибровочной симметрии привела к полному определению лагранжианов взаимодействия для электромагнитных, слабых, сильных и гравитационных взаимодействий и создала иллюзии о построении «теории всего». Однако, так же, как и в классической физике, стало ясно, что в основе дедуктивного получения решений (законов природы) лежат не только принципы симметрии лагранжианов. Для однозначного определения решения необходимы *дополнительные условия*, без которых решения уравнений Лагранжа неоднозначны. Дополнительные условия: предположения о константах, входящих в лагранжианы, интегральные симметрии решений, краевые и начальные условия и т.п., столь существенны, что в ряде случаев можно конструировать модели (решения, законы природы), не опираясь на лагранжев метод. В качестве примера приводится использование такого подхода в одном из наиболее бурно развивающихся разделов современной физики — релятивистской ядерной физике. Точный математический язык калибровочной симметрии — дифференциальная геометрия, а точный язык для дополнительных условий — топология, свойства пространства параметров в целом. В настоящей статье отмечается фундаментальный вклад В.А.Фока в разработку понятия пространства — первичного понятия физики.

At the beginning of the ’50s, at an all-union conference, Vladimir Alexandrovich Fock presented a large talk on the theory of gravitation. During the discussion, a very competent physicist said that Fock in his talk gave the well-known Einstein equations (by implying the absence of novelty). Then Fock replied: «I have known your philosophy for a long time: if the equations are the same, then the theory is also the same». As we presently know, at that time V.A.Fock was working on a fundamental monograph «The Theory of Space, Time and Gravitation» [1]. In the preface to the first edition of the book he writes: «The results of these investigations have led us to the conviction that, at least for the most important class of physical problems, it is possible to obtain unambiguous solutions for the gravitation equations by imposing additional conditions compatible with them. This idea has underlain a new point of view on the whole of the theory of gravitation. Therefore, there arises the necessity of formulating the whole of the theory of space, time and gravitation from this newly elaborated point of view, which has just been done in this book».

Fock's point of view on the theory of relativity and the theory of gravitation was after all generally recognized. Fock stressed that the theory of gravitation and, generally speaking, any theory cannot be formulated by confining oneself to the local consideration. It is necessary to consider the «space as a whole», its global structure, and its topology. Otherwise, it is impossible to formulate the problem unambiguously. The equations of any field are the equations in partial derivatives the solutions of which are unambiguous only in the presence of initial, boundary and limiting conditions.

The laws of Nature are relations between invariants, as far as they should not be dependent on symmetry transformations. The hypothesis about the symmetry possessed by a system are axioms determining the state of the system and its behaviour. Starting from symmetry principles it is possible to derive new laws of Nature deductively, and not only by observing physical objects or solving equations. Weyl wrote that as far as he could judge, all *a priori* ideas in physics have a symmetry origin. The symmetry of the «space as a whole» essentially supplements the symmetry and invariance defining the Lagrangian density, and, in many cases, enables us to construct models (solutions, laws of Nature) starting from the first principles, not using the Lagrange method.

For a long time, mathematicians have paid attention to the integral invariants in topology and to the connection between differential geometry and the theory of surfaces. This trend was initiated by the famous Gauss-Bonnet theorem which says that the integral of the Gaussian curvature over an entire surface is a topological invariant and is integer multiple of 2π. For a sphere, no matter how distorted, the integral curvature is 4π, for a torus it is zero, while for the «double-holed torus» it is 4π, and so on. The Gaussian curvature is a local parameter. It can be measured by measuring the angles and the sides of small triangles. For example, to show the Earth to be round it is not necessary to circumnavigate the globe and to take photographs from outer space. Eratosphenes did it by comparing shadows in Alexandria and Syene.

Auxiliary spaces are useful in studying ordinary surfaces and their higher-dimensional analogs. One example is the space consisting of the tangent planes to a surface. Such spaces are called «fiber bundles». The «fibers» are the auxiliary spaces — the tangent planes. The fiber bundles are an appropriate framework for gauge theories, developed to deal in a unified way with electromagnetic, weak and strong interactions.

Noting V.A.Fock's contribution to contemporary physics it is necessary to stress (this was also emphasized by Chen Ning Yang, one of the principal architects of the gauge theory) that the gauge theory is a generalization of «the gauge symmetry in electromagnetism known from the papers by Fock and Weyl».

In the 1930s and 1940s L.S.Pontrjagin and other mathematicians have found, without undergoing the influence of physical models, interesting topological invariants playing an ever-growing role in modern physics. The integral geometry makes it possible to study classical solutions for gauge fields. The merging of the newest areas of mathematics and theoretical physics enables us to hope that along this way one will succeed in finding methods for obtaining nonperturbative solutions of the Lagrange equations for gauge fields. The nonperturbative methods in the *Standard Model* take one of the central places in modern theoretical physics. Among them of special interest are multiboson processes in electroweak physics. These phenomena are associated with the violation of the sum of the baryon (B) and the lepton (L) numbers in the *Standard Model* [3]. Therefore such processes determine the evolution of $(B+L)$ at high temperature in the early universe, that is, the origin of the baryons — baryosynthesis. Also specific calculations show [5] that the processes with $(B+L)$ violation and production of many electroweak bosons might be *in principle* observable in collisions at energies higher than 18 TeV. The initial and the final states containing many bosons (many→ many scattering) are described by quasi-classical methods with the use of nontrivial classical solutions of the field theory periodic — instantons. The $(B+L)$ violation is caused by tunnel transitions between the states with different topological charges q for the electroweak gauge fields and is described by the formula:

$$\Delta(B+L) = 6q.$$

The peaks of these potential barriers — sphalerons — (the energy E is ordinarily plotted against q) are, in the order of magnitude, equal to $m_w/\alpha_w \sim 10$ TeV, where m_w is the mass of an intermediate boson, and α_w is the electroweak interaction constant. The treatment of the auxiliary conditions describing the initial states of multiple interactions with the use of the topological properties of gauge fields has resulted in fundamental conclusions for elementary particle physics, cosmology [4] and even for designing a new generation of accelerators at superhigh energies [6].

In 1931, when solving the one-dimensional Heisenberg model of a ferromagnet, Bethe [7] formulated a hypothesis about a wave function of the model. In 1967 Ch.N.Yang has generalized [8] this hypothesis by imposing on matrices $A(u)$ and $B(v)$, which occurred in the development of the hypothesis, the following conditions:

$$A(u) \cdot B(u+v) \cdot A(v) = B(v) \cdot A(u+v) \cdot B(u). \tag{1}$$

Many one-dimensional quantum mechanical problems, in which the Bethe hypothesis is valid, are known. In each case, the consistency condition is Eq. (1), where the operators $A(u)$ and $B(v)$ and the one-dimensional coordinates u and v take different forms in different problems. During the past ten or fifteen years, a large number of developments in physics and mathematics have led to the conclusion that Eq. (1) is a fundamental mathematical structure. Equation (1) has the generally accepted name — the Yang–Baxter equation.

Ch.N.Yang shows [9] how Eq. (1) affected modern physics and mathematics:

«**Physics**:

- One-dimensional quantum mechanical problems
- Two-dimensional classical statistical mechanical problems
- Conformal field theory

Mathematics:

- Knot theory, braid theory
- Operator theory
- Kopf algebra
- «Quantum groups»
- Topology of 3-manyfold
- Monodromy of differential equations

There is an explosion of literature on these subjects. In order to find these, one could consult the three recent review volumes and reprint collections listed in the footnote*.

Why does the Yang–Baxter equation enter into so many different areas of mathematics and physics? I believe the answer is that the equation is a kind of generalization of the structure of the permutation group».

*120 Braid Group, Knot Theory and Statistical mechanics, eds. C.N.Yang and M.L.Ge (World Scientific, Singapore, 1989); Yang–Baxter Equation in Integrable Systems, ed. M.Jimbo (World Scientific, Singapore, 1990); Yang–Baxter Equations, Conformal Invariance and Integrability in Statistical Mechanics and Field Theory, eds. B.Barber and P.Pearce (World Scientific, Singapore, 1990).

A particular role of the one-dimensional problem is due to the possibility of establishing a definite order in the particle disposition in a one-dimensional space.

As a hypothesis about the properties of the solutions of statistical physics N.N. Bogolubov has formulated the correlation depletion principle [10]. The principle is based on the intuitive idea that the correlation between spatially separated groups of particles of a macroscopic system practically vanishes. The correlation depletion principle was successfully applied to the development of the theory of ferromagnetism, superfluidity and superconductivity. Also it is possible to formulate the notion of quasi-averages and the properties of the solutions that afterwards were given the name of spontaneous symmetry breaking.

It is interesting that the well-known attempt of Dirac to formulate a relativistic theory of dynamical systems [11] led him to the realization that it was possible to state only the necessary but not the sufficient conditions for this theory to exist. At the end of his remarkable article, Dirac writes, «Some further condition is needed to ensure that the interaction between two physical objects becomes small when the objects become far apart. It is not clear how this condition can be formulated mathematically». The correlation depletion principle of Bogolubov is formulated as an asymptotic form of the Green functions as universal (independent of the specific features of the system) linear forms from averages of the product of field functions. This principle gives mathematical formulation for the additional condition of the relativistic theory (Poisson's brackets) developed by Dirac.

In Refs. 12, 13 the correlation depletion principle is formulated both in relative 4-velocity space and the Lobachevsky space. The application of this principle to quantum chromodynamics of large distances (or, more precisely, of small relative velocities), to the description of multiple production processes, and, particularly, to relativistic nuclear physics was found to be especially productive. In these areas, the perturbative approach does not work, thus hypotheses of a fundamental character, i.e., auxiliary conditions, are needed. A collision of relativistic nuclei results in the production of many particles, and the interaction picture is very complicated. Both nucleon and quark-gluon degrees of freedom participate in the same collision. The number of the parameters of the problem is extremely large, and it is particularly important to discover the invariants [13]. Relativistic nuclear physics that was born at the beginning of the '70s in Dubna became one of the most intensively developed areas of high energy physics in many laboratories of the world.

The discovery of the laws of relativistic nuclear physics is a part of the general search for the laws describing relativistic multiparticle systems, including macroscopic systems. These problems were studied by outstanding scientists of the 20th century. The first studies were devoted to the transport equations which allowed the formulation of the thermodynamical properties of dilute relativistic multiple systems. The great success of quantum field theory in describing multiparticle systems on the basis of the Hamiltonian method has not resulted however in great progress in the development of the problems of relativistic nuclear physics. In Refs. 12, 13, it is shown that the approach to relativistic nuclear physics based on the geometry of velocity space and hypotheses about the asymptotic nature of the laws in this space allows us to put in order an enormous amount of experimental data and make quantitative predictions. Some of these predictions make many experiments on huge accelerators unnecessary and even condemned to failure. The methods of symmetry of the solutions utilized in these papers are analogous to the methods of the mechanics of continuous media and consist of the following:

1. The parameters describing the problem — the space defining parameters — are selected.
2. The symmetry of this space is seen or guessed, and the corresponding invariants are determined.
3. The laws of Nature are treated as relations between invariants. The mathematical language of symmetry — group theory — is especially effective here.
4. Additional principles — the correlation depletion principles, the intermediate asymptotics, the hypothesis of the analyticity of physical laws are used.

In the case of relativistic nuclear physics, the defining parameters are the cross sections, quantities derived from them, and the invariant dimensionless intervals in relative 4-velocity space $\mathbf{u}_i = \mathbf{p}_i/m_i$; $u_i^0 = E_i/m_i$:

$$b_{ik} = -(u_i - u_k)^2 = 2[(u_i \cdot u_k) - 1] = 2\left[\frac{E_i \cdot E_k - \mathbf{p}_i \cdot \mathbf{p}_k}{m_i \cdot m_k} - 1\right].$$

As far as the energies E_i and the momenta $\mathbf{p}_i$ are linked by the known relation $E_i^2 - \mathbf{p}_i^2 = m_i^2$, then $(u_i)^2 = (u_0)^2 - (\mathbf{u}_i)^2 = 1$. Instead of 4-dimensional space it is possible to introduce 3-dimensional one, with the 4th coordinate expressed in terms of the other three:

$$u_i^0 = \pm\sqrt{1 + u_x^2 + u_y^2 + u_z^2}. \tag{2}$$

This equation is a two-sheeted hyperboloid. The geometry on the surface of the hyperboloid is the geometry of 3-dimensional Lobachevsky space, analogous to the geometry on the surface of a sphere. The interval between points on the surface of a sphere is given by the cosine of the angle of the great circle, and the interval on the surface of the hyperboloid is given by the hyperbolic cosine of the rapidity

$$\rho = \frac{1}{2}\ln\frac{E + |\mathbf{p}\,|}{E - |\mathbf{p}\,|}.$$

The relation between the intervals b_{ik} and ρ_{ik} is of the form:

$$b_{ik} = 2[(u_i \cdot u_k) - 1] = 2[ch\rho_{ik} - 1].$$

The number of the parameters of b_{ik} is $n(n-1)/2$. The most complete description of the final states of nuclear collisions is connected with the use of triangulation and the construction of polyhedra in velocity space.

The introduction of the variable N_I and N_{II} characterizing the effective numbers of nucleons participating in the collisions of nuclei I and II has proved very productive. In a wide interval of relative velocities the additional variables N_I and N_{II} turned out to be continuous and smooth. The invariant that is employed to express a large number of the laws of relativistic nuclear physics has the meaning of the minimal mass

$$\min[m_0^2(u_I N_I + u_{II} N_{II})^2]^{1/2} = 2m_0\Pi$$

under the condition of conservation of the 4-momentum:

$$m_0 u_I N_I + m_0 u_{II} N_{II} = \sum_i p_i.$$

Here U_I and U_{II} are the 4-velocities of the nucleus as a whole, m_0 is the mass of one nucleon. The introduction of the single self-similarity parameter (invariant)

$$\Pi = \frac{1}{2}\sqrt{(u_I N_I + u_{II} N_{II})^2}$$

allowed a quantitative description of the cumulative effect, deep-subthreshold, near-threshold phenomena, and antimatter production in nucleus-nucleus collisions [14].

The equation

$$E\frac{d^3\sigma}{d\mathbf{p}} = C_1 A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot f(\Pi), \tag{3}$$

where A_I and A_{II} are the atomic weights of colliding nuclei, C_1 is the constant, describes a variety of nuclear reactions as the cross section changes by eight orders of magnitude. However, to find the limits of the parameter space, where the description of physical processes on the basis of this model becomes invalid, some auxiliary work is needed. Of special interest is the prediction [15] on this basis of the results of projected experiments on presently designed nuclear colliders. For collider energies the interval between the points I and II is:

$$(u_I \cdot u_{II}) = ch\rho_{I\,II} >> 1.$$

The relation between the sides of the Lobachevsky triangle is of the form:

$$(u_I \cdot u_k) = u_I^0 \cdot u_k^0 - \vec{u}_I \cdot \vec{u}_k = ch\rho_{I\,k} =$$
$$= ch\rho_{I\,II} \cdot ch\rho_{II\,k} - sh\rho_{I\,II} \cdot sh\rho_{II\,k} \cdot \rho_{II\,k} \cdot \cos\theta_{I\,k} \approx$$
$$\approx ch\rho_{I\,II}(ch\rho_{II\,k} - sh\rho_{II\,k} \cdot \cos\theta_{I\,k}) = ch\rho_{I\,II} \cdot x_k.$$

Here x_k is a known light cone variable. At large relative velocities Eq. (2) turns into the light cone equation which is used in constructing models of high energy physics in velocity space.

After discovering non-Euclidean geometry Lobachevsky posed the problem of using it to describe real physical phenomena. The hypothesis that at large distances the relations between the sides and angles of triangles might satisfy a new geometry was not confirmed by his analysis of astronomical data.

V.A.Fock demonstrated the validity of the Lobachevsky geometry in relative velocity space by considering the phenomenon of astronomical aberration [1]. The phenomenon is that in two moving relatively each other frames of reference the directions to the same star do not coincide, but differ by the magnitude of aberration. To find this value it is necessary to construct a Lobachevsky triangle with vertices in the points $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3 = \mathbf{a}C$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the bodies to which both the frames are related. Here $\mathbf{a}$ is a unit vector in the direction of the light wave going from the star. In astronomical observations visible positions of a star are compared for different directions of motion of the globe along the orbit (annual aberration).

By analyzing the concept of physical space, Fock stresses that this primary concept is obtained by means of appropriate abstractions of space-time relations between material processes. The relations are established on the basis of the hypothesis about applicability of the Euclidean geometry to a real physical space as well as on the suppositions about the existence of solid bodies and rectilinear propagation of light.

Thus, the properties of the light and those of solid bodies (distance measurements) play the fundamental part in establishing the geometry of a real physical space. Another feasible way of determining the location of objects in space, which in principle differs from triangulation, is radiolocation or radiogeodesy. However, in any case, the definition of the concept of physical space depends on precision of measuring procedures. The correspondence of it to the mathematical concept of space requires some reservations. Fock also notes that the terms «the space as a whole», «conditions at infinity», and so on, are employed by him in the mathematical sense admitted in field theory. The space as a whole implies an area which is large enough for the field induced by a body system to be negligible on its boundaries. Depending on the character of the problem the dimensions of the area are very different. A micron compared with the atom may be thought of as an infinitely large quantity, the light year for the Sun system and billions of light years for galaxy accumulation are infinitely large quantities. When formulating a theory, new generalizations are introduced, as a result of which the law may become approximate, but this does not diminish its fundamental importance.

Complicated real physical situations require simplified descriptions by means of symbolic, and even verbal, models based on experimentally testable hypotheses. However the correspondence of the physical space to the mathematical one appears not only as a result of generalization of experience and measuring procedures. The correspondence of the velocity space to the Lobachevsky space is a result of a deduction. More striking example is the introduction by Einstein of the Riemann space in the theory of gravitation which is of a particularly deductive nature. As Fock notices, this requires that the properties of the «space as a whole» should be considered. Otherwise, it is impossible to formulate the problem in an unambiguous manner. V.A.Fock analyses various suppositions and gives much attention to the theory of a space homogeneous at infinity. He attaches great importance to the possibility of introducing in this case a privileged frame of reference determined with an accuracy up to the Lorentz transformation (harmonic coordinates). All concrete problems of the theory of gravitation are solved in Ref. 1 in harmonic coordinates.

Special attention should be paid to Fock's formulation of the Hilbert space in quantum theory of radiations. In Ref. 16 Fock notices that the mathematical apparatus of quantum theory of emission and absorption of photons created by Dirac does not correspond to the physics of this phenomenon and suggests a mathematical basis of the theory. In his book «The Principles of Quantum Mechanics» Dirac called it «Fock's representation». Finally «Fock's space» suggested for mathematical description of the systems with an interaction Lagrangian changing the particle number became the generally recognized concept of quantum field theory. It is of far reaching importance in present-day applications of quantum chromodynamics, in particular, in quark-parton model.

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