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RELATIVISTIC MULTIPARTICLE PROCESSES IN THE CENTRAL RAPIDITY REGION AT ASYMPTOTICALLY HIGH ENERGIES

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The principles of symmetry and self-similarity have been used to obtain an explicit analytical expression for inclusive cross sections of production of particles, nuclear fragments and antinuclei in relativistic nuclear collisions in the central rapidity region ($y = 0$). The result is in agreement with available experimental data. It is shown that the effective number of nucleons participating in nuclear collisions decreases with increasing energy and the cross section tends to a constant value equal both for particles and for antiparticles. The analysis of the obtained results makes it possible to conclude that the hopes for obtaining dense and hot matter in heavy unltrarelativistic nuclear collisions will not be realized.

The investigation has been performed at the Laboratory of High Energies, JINR.

Релятивистские многочастичные процессы в центральной области быстрот при асимптотически высоких энергиях

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На основе принципов симметрии и подобия найдено явное аналитическое выражение для инклюзивных сечений образования частиц, ядерных фрагментов и антиядер при столкновении релятивистских ядер в центральной области быстрот ($y = 0$). Результат согласуется с имеющимися экспериментальными данными. Показано, что эффективное число нуклонов, участвующих в столкновении ядер, с ростом энергии уменьшается, а сечение стремится к постоянной величине, равной для частиц и античастиц. Анализ полученных результатов позволяет сделать заключение, что надежды на получение плотной и горячей материи при столкновении тяжелых ультрарелятивистских ядер на оправдаются.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

It has become common use to consider that the collisions of highly relativistic nuclei can create quasi-macroscopic systems consisting of quarks, gluons and nucleons. It is expected that in such collisions one will succeed in obtaining hot and dense nuclear matter and in observing phase transitions between hadronic matter and quark-gluon plasma. The phase transitions are the most typical and essential phenomena of the collective behaviour of particle systems. Quantum chromodynamics describes quarks and gluons and, in principle, must describe their collective behaviour under appropriate boundary and initial conditions. For a strong compression, when the distances between colour objects (quarks, gluons) decrease, the potential looks like the Coulomb one, and the system consisting of quarks and gluons becomes similar to electron-ion plasma. This is also confirmed by lattice calculations in the framework of quantum chromodynamics.

The models in which nuclear matter is thought of as a continuous medium at high pressure and temperatures are of great interest to historians of the early Universe who are trying to reconstruct the first microseconds of the world creation. The nuclear equation of state gives very important information necessary for quantitative understanding of astrophysical phenomena such as explosion of supernova and formation of neutron stars as well as for describing their characteristics.

However the question of the extent to which even the heaviest nuclei at the highest available and planned energies can imitate continuous medium remains still open. Besides, it is not clear to what extent the laws of hadron thermodynamics applied to the description of the early Universe, interstar processes and so on are valid for the dynamics of multihadron systems studied at accelerators.

In the models under consideration the multiparticle (internal and intermediate) states are described in terms of macroscopic variables (temperature, pressure, density, ...) which are not observable in the study of relativistic nuclear collisions. In the present paper, using the approaches based on the application of the laws of symmetry and self-similarity and the correlation depletion principle in relativistic nuclear physics we answer the following questions:

1. What is the effective number of nucleons, that participate in nucleus-nucleus collisions, depending upon the measured parameters of inclusive particles?

2. What is the asymptotic behaviour of the effective number of nucleons depending upon the collision energy?

The answers to these questions should show that the highest energies and the largest atomic weights of colliding particles make it impossible to obtain quark-gluon plasma in laboratory conditions. The regularities in the behaviour of the cross sections in the central rapidity region we have observed are tested using the available in literature experimental material and predictions for the collider energy region of nuclear collisions have been made.

At the Joint Institute for Nuclear Research, starting with the first papers on cumulative effect [1], the notion of the minimal number of nucleons participating in the nucleus-nucleus collision

$$I + II \rightarrow 1 + 2 + \ldots \qquad (1)$$

was introduced. There these quantities were denoted as N_I and N_{II}. The quantity $N^{\min}$ corresponds to the minimal number of nucleons which should take part in the transfer of the momentum to an observed fast particle according to the laws of conservation.

The cumulative effect is defined as the production of particles in the region

$$(u_1 u_{II}) > (u_I u_{II}) >> 1,$$

when

$$N \geq N^{\min} = \frac{m_1}{m_0} \frac{(u_1 u_{II})}{(u_I u_{II})} > 1, \qquad (2)$$

where

$$u_i = p_i / m_i = \left\{ \frac{E_i}{m_i} ; \frac{\mathbf{p}_i}{m_i} \right\},$$

E_i is the energy, $\mathbf{p}_i$ and m_i are the three-dimensional momentum and the mass of an i-th particle, respectively, m_1 is the inclusive particle mass, m_0 is the nucleon mass.

Generalization of these ideas given in Ref.2 consists in the introduction of a self-similarity parameter

$$\Pi = \min \left[\frac{1}{2} \sqrt{(u_I N_I + u_{II} N_{II})^2} \right]. \tag{3}$$

The quantities N_I and N_{II} become measurable if we take into account the law of conservation of four-momentum in the form:

$$(N_I m_0 u_I + N_{II} m_0 u_{II} - m_1 u_1)^2 = (N_I m_0 + N_{II} m_0 + \Delta)^2 \tag{4}$$

neglecting the relative motion of all the remaining not detected particles.

Δ is the mass of the particles providing conservation of the baryon number, strangeness and other quantum numbers. For antinuclei and K^- mesons (the case of antimatter formation [2]) $\Delta = m_1$, for nuclear fragments $\Delta = -m_1$. For particles produced without accompanying antiparticles (π mesons, jets and others) $\Delta = 0$.

Eq. (4) can be written in the form

$$N_I N_{II} - \Phi_I N_I - \Phi_{II} N_{II} = \Phi_\delta , \tag{5}$$

where we introduce relativistic invariant dimensionless quantities:

$$\Phi_I = \left[\frac{m_1}{m_0} (u_I u_1) + \frac{\Delta}{m_0} \right] / [(u_I u_{II}) - 1],$$

$$\Phi_{II} = \left[\frac{m_1}{m_0} (u_{II} u_1) + \frac{\Delta}{m_0} \right] / [(u_I u_{II}) - 1],$$

$$\Phi_\delta = (\Delta^2 - m_1^2) / [2m_0^2((u_I u_{II}) - 1)].$$

We write Eq. (5) in the form

$$[(N_I / \Phi_{II}) - 1] \cdot [(N_{II} / \Phi_I) - 1] = 1 + [\Phi_\delta / (\Phi_I \Phi_{II})] \tag{6}$$

and find the self-similarity parameter Π as a solution of the equations

$$d(u_I N_I + u_{II} N_{II})^2 / dF_I = 0,$$

$$d(u_I N_I + u_{II} N_{II})^2 / dF_{II} = 0.$$

Here we introduce the variables

$$F_I = (N_I/\Phi_{II}) - 1,$$

$$F_{II} = (N_{II}/\Phi_I) - 1, \tag{7}$$

$$F_I \cdot F_{II} = 1 + [\Phi_\delta/(\Phi_I \Phi_{II})].$$

These equations are symmetric with respect to the replacement of F_I by F_{II} and Φ_I by Φ_{II} and for $\Phi_I = \Phi_{II}$ they coincide. From here for the minimal value of Π we have the following solution valid for $(u_1 u_I) = (u_1 u_{II})$, i.e., in the central rapidity region:

$$F_I = F_{II} = F; \quad F^2 = 1 + (\Phi_\delta/\Phi^2).$$

Making use of Eq.(7) we get $(N_I/\Phi) - 1 = (N_{II}/\Phi) - 1$ and, consequently, $N_I = N_{II} = N$. Thus, we obtain

$$N = N_I = N_{II} = (1 + F)\,\Phi = [1 + [1 + (\Phi_\delta/\Phi^2)]^{1/2}]\,\Phi,$$

$$\Pi = \frac{1}{2}\sqrt{2N^2 + 2N^2(u_I u_{II})} = \frac{N}{\sqrt{2}}\sqrt{1 + (u_I u_{II})} = N\cosh Y, \tag{8}$$

$$\Pi = N\cosh Y. \tag{9}$$

Taking into account that

$$(u_I u_{II}) = \cosh(2Y)$$

$$(u_I u_1) = \frac{m_T}{m_1}\cosh(-Y - y) = \frac{m_T}{m_1}\cosh(Y + y)$$

$$(u_{II} u_1) = \frac{m_T}{m_1}\cosh(Y - y)$$

at $y = 0$ we get

$$(u_I u_1) = (u_{II} u_1) = \frac{m_T}{m_1}\cosh Y.$$

Here

$$m_T = \sqrt{m_1^2 + p_T^2}.$$

Then

$$\Phi = \frac{1}{m_0}[m_T\cosh Y + \Delta](1/2\sinh^2 Y),$$

$$\Phi_\delta = (\Delta^2 - m_1^2)/(4m_0^2 \sinh^2 Y). \tag{10}$$

From Eq.(8), by using Eq.(10) we get

$$N = [1 + \sqrt{(\Phi_\delta/\Phi^2) + 1}\,]\left[\frac{m_T}{m_0}\cosh Y + \frac{\Delta}{m_0}\right][1/(2\sinh^2 Y)]. \tag{11}$$

Now we consider the asymptotic behaviour of the self-similarity parameter with increasing interaction energy. Employing Eqs.(10) we obtain at

$$s/(2m_I m_{II}) \approx (u_I u_{II}) = \cosh 2Y \to \infty$$

$$\Phi_\delta/\Phi^2 = \frac{\Delta^2 - m_1^2}{m_T^2}\,\frac{\sinh^2 Y}{[\cosh Y + (\Delta/m_T)]^2} \to \frac{\Delta^2 - m_1^2}{m_T^2},$$

$$\Phi \cosh Y = \left(\frac{m_T}{m_0}\cosh Y + \frac{\Delta}{m_0}\right)\frac{\cosh Y}{2\sinh^2 Y} \to \frac{m_T}{2m_0}.$$

Hence it follows that at $\cosh Y \to \infty$ (in the collider energy region) the self-similarity parameter Π assumes the finite value

$$\Pi_\infty = \frac{m_T}{2m_0}[1 + \sqrt{1 + (\Delta^2 - m_1^2)/m_T^2}\,]. \tag{12}$$

As is seen from Eq.(11), the effective number of the nucleons involved in the reaction $N \to 0$ at $\cosh Y \to \infty$. In this connection, we may say with certainty that the hopes for obtaining dense and hot matter (in any case, for detecting it by fast inclusive particles) in ultrarelativistic nuclear collisions are not feasible. Our conclusion is based on earlier carried out tests of the dependence of the invariant differential cross section upon the self-similarity parameter Π:

$$\frac{d^2\sigma}{m_T\, dm_T\, dy} = 2\pi C_1 A_I^{1/3 + N_I/3} A_{II}^{1/3 + N_{II}/3} \exp[-\Pi/C_2], \tag{13}$$

where $C_1 = 1.9 \cdot 10^4$ mb $\cdot$ GeV$^{-2}C^3\cdot$ sr^{-1}, $C_2 = 0.125 \pm 0.002$ in a wide range of variables y, m_T, $\cosh Y$ and for different inclusive particles [3].

An additional possibility of testing Eq.(13) appeared as a result of the CERN SPS experiments on Pb + Pb collisions at an energy of 160 A $\cdot$ GeV which corresponds to $\cosh Y = 8.971$. It is especially interesting to estimate the effect of the transition to heavy nuclei on the number of the nucleons in the initial state participating in production of inclusive particles.

Predictions of the cross sections for antimatter production in relativistic nuclear collisions and, in particular, quantitative predictions of the cross sections for the reactions

$$\mathrm{Pb} + \mathrm{Pb} \to \bar{p} + \ldots \quad \text{and} \quad \mathrm{Pb} + \mathrm{Pb} \to \bar{d} + \ldots$$

at an energy of 160 A · GeV are given in Ref. 2 over the whole rapidity region. However a computor method for solution of the equations

$$\frac{d\Pi}{dN_I} = 0 \quad \text{and} \quad \frac{d\Pi}{dN_{II}} = 0$$

suggested there makes it possible to obtain the results in the form of numerical tables and diagrams. The analytical representation for Π enables us to draw the following new conclusions:

1. There exists the limiting value of Π described by Eq.(12).

2. For $\Phi_\delta = 0$ the expression for Π is factorized and proportionality of m_1 to the inclusive particle mass makes it possible to test in detail the self-similarity laws. From Eqs.(11) and (9) it follows that the cross section (13) exponentially quickly decreases with increasing m_1. In particular, this implies that the probability of observing even light antinuclei and fragments in the region $y = 0$ is insignificantly small.

3. A relative yield of strange particles in the central region (for $y = 0$) increases with increasing collision energy $\cosh 2Y$. For example, in the asymptotic region, i.e., for $\cosh 2Y \to \infty$, $K^- / p >> 1$.

4. The effective number of nucleons involved in the reaction decreases with increasing $\cosh Y$ (Eq.(11)).

5. A strong factorizable dependence of Π on $m_T = \sqrt{m_1^2 + p_T^2}$ we have discovered explains the observed m_T scaling.

At $p_T = m_T / m_0 \geq 5$ (see Eqs.(9),(11)) the cross section becomes very small and the transition from the exponential dependence to the power one (hard scattering) should be observed.

We give a number of estimates of the validity of the above-mentioned equations on the basis of the available in literature experimental data. We consider the ratio of the yield of antideuterons to that of deuterons at an energy of 160 GeV per nucleon in the collisions of Pb nuclei which have been realized at the SPS.

$$(u_I u_{II}) = \cosh 2Y = E_I / m_I \approx 160\ \mathrm{GeV} / m_0 \approx 160,$$

$$2Y = 5.7683, \quad Y = 2.884,$$

$$\cosh Y = 8.971, \quad \sinh Y = 8.915.$$

For d and anti-d (and, generally, nucleus-antinucleus) $\Delta^2 - m_1^2 = 0$. Let us consider the case $m_T = m_1$ ($p_T = 0$) corresponding to the upper estimate of the cross section (13). Then for the deuteron we have

$$\Pi_d = \left[\frac{m_1}{m_0}\cosh Y - \frac{m_1}{m_0}\right]\frac{\cosh Y}{\sinh^2 Y} = 1.799;$$

and for the antideuteron,

$$\Pi_{\bar{d}} = \left[\frac{m_1}{m_0}\cosh Y + \frac{m_1}{m_0}\right]\frac{\cosh Y}{\sinh^2 Y} = 2.251.$$

By inserting these numbers in Eq.(13) we find the ratio of the production cross section for antideuterons and deuterons

$$\bar{d}/d = \exp\left[-(\Pi_{\bar{d}} - \Pi_d)/C_2\right] = 0.027.$$

In the asymptotical region $\Pi_{\bar{d}} = \Pi_d$ (according to Eq.(12)) and the ratio of the production cross section for deuterons to that for antideuterons is close to unity. Similar estimates for K^-/K^+ and $\bar{p}/p$ give:

$$K^-/K^+ \approx 0.25; \quad \bar{p}/p \approx 0.16.$$

Preliminary data of the NA52 and NA44 experiments on measurements of the yields of antiparticles produced as a result of the interactions of Pb nuclei of an energy of 160 GeV per nucleon with a Pb target have recently been obtained at the CERN SPS [4,5]. In these papers, in particular, the rapidity distributions for the ratios of the $\bar{p}/p$, K^-/K^+ and $\bar{d}/d$ yields have been presented. The ratios for the particle and antiparticle yields in the central rapidity region $y = 0$ measured in these experiments are in agreement with our above-mentioned upper estimates or are close to the latter in magnitude. For the sake of comparison the results of our estimates and the experimental values are tabulated:

Ratios of the yields	$\bar{p}/p$	$\bar{d}/d$	K^-/K^+
Calculation (the present paper) $p_T = 0$	0.16	0.027	0.25
NA52	≈ 0.1	≈ 0.01	≈ 0.2
NA44	≈ 0.08	—	≈ 0.4

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