7.C

Nuclear Physics **18** (1960) 310—317; © *North-Holland Publishing Co., Amsterdam*

POLARIZABILITY OF NUCLEONS

A. M. BALDIN

P. N. Lebedev Physical Institute, Academy of Sciences, Moscow, USSR

Received 18 December 1959

Abstract: Estimates of dipole polarizabilities of nucleons and the values they involve are given on the basis of data on photo-production of π-mesons and the Compton effect on nucleons. It is indicated that no upper estimate of neutron polarizability exists at present. The preliminary experimental data now available may be interpreted as indicating that a neutron has an abnormally large polarizability. The effects leading to the inapplicability of the impulse approximation for describing the reaction $\gamma+d \rightarrow p+n+\gamma'$ are estimated. It is pointed out that the measurement of the cross section of the reaction $\gamma+d \rightarrow d+\gamma'$ would yield an answer for the value of neutron dipole polarizability †.

1. Introduction

As is well known from classical electrodynamics, the interactions of photons with a system of charges may be described by a fairly small number of real parameters provided the wavelength of the photon considerably exceeds the dimensions of the system and the photon's frequency is essentially less than the resonance frequencies of the system. These constants fully determine the behaviour of the system in static or slowly changing fields and are expressed through the charge, the magnetic moment and electric and magnetic polarizability tensors.

These characteristics may in particular be considered for the nucleon [1]). Of especial interest are the electric and magnetic polarizabilities of the nucleon since these are its "structural" characteristics. The aims of the present paper are: a) determination of the frequency region of photons for which the interaction of a nucleon and electromagnetic field may be described by four constants (charge, magnetic moment, electric and magnetic polarizability) with satisfactory accuracy (say, 5 %); b) discussion of possible estimates of the magnitude of the polarizability on the basis of the available experimental data and discussion of the experiments which might be worthwhile for specifying polarizability quantities.

† The main part of this paper was reported at the Padua-Venice Conference, September 1957. It was not included into the Conference Proceedings because of technical difficulties. In view of the recent interest in these estimates (see refs. [5-8])) we decided to publish them together with certain additional considerations about the Compton effect on deuterium and neutron polarizability.

2. Definition of Polarizability

In ref. [1]) it was shown that if the scattering matrix is written as

$$\hat{S} = -e'_n \hat{g}_{nj} e_j (4\omega\omega')_{\frac{1}{2}}$$

(here and in the following summation over dummy indices n, $j = 1, 2, 3$ is understood; e_n and e'_j are the components of the polarization vectors of the incident and scattered photons, while ω and ω' are their frequencies), the following theorem holds in virtue of gauge invariance:

$$k'_n g_{nj} k_j = \omega' \omega \int \mathrm{d}x\mathrm{d}y \exp[-ik'x] \exp[iky] \mathrm{P}[\hat{\varrho}(x), \hat{\varrho}(y)].$$

Here $k'x$ and ky are scalar products of 4-vectors, k and k' being the 4-vectors of the photon momentum, $\hat{\varrho}(x)$ is the charge density operator, and P Dyson's time ordering operator. For purposes of orientation we may note that for Thomson scattering g_{nj} has the form:

$$\hat{g}^{\mathrm{T}}_{nj} = \delta_{nj}(2\pi)^4 i\delta(\Delta k + \Delta p)\,\frac{e^2}{M},$$

where the δ-function expresses the law of 4-momentum conservation.

We may now expand the mean value $\langle f|k_n g_{nj} k_j|0\rangle$ into a sum over states, where $|0\rangle$ and $|\mathrm{f}\rangle$ are the unexcited states of the nucleon:

$$\langle \mathrm{f}|k'_n \hat{g}_{nj} k_j|0\rangle = \omega'\omega \frac{(2\pi)^4}{i}\,\delta(\Delta k + \Delta p)$$

$$\times \sum_{\mathrm{N}} \left\{ \frac{\langle \mathrm{f}|\int \hat{\varrho}(\mathbf{x}) \exp[-i\mathbf{k}'\cdot\mathbf{x}]\mathrm{d}\mathbf{x}|\mathrm{N}\rangle\langle \mathrm{N}|\int \hat{\varrho}(\mathbf{y}) \exp[i\mathbf{k}\cdot\mathbf{y}]\mathrm{d}\mathbf{y}|0\rangle}{E_{\mathrm{N}} - E_0 - \omega} \right.$$

$$\left. + \frac{\langle \mathrm{f}|\int \hat{\varrho}(\mathbf{y}) \exp[i\mathbf{k}\cdot\mathbf{y}]\mathrm{d}\mathbf{y}|\mathrm{N}\rangle\langle \mathrm{N}|\int \hat{\varrho}(\mathbf{x}) \exp[-i\mathbf{k}'\cdot\mathbf{x}]\mathrm{d}\mathbf{x}|0\rangle}{E_{\mathrm{N}} - E_0 + \omega'} \right\}.$$

Hence it is easy to obtain the terms of zero and first orders in the expansion of the scattering amplitude with respect to the photon frequency (see ref. [1]).

The next term of the expansion $\langle \mathrm{f}|k'_n g_{nj} k_j|0\rangle$ involving excited intermediate states has the form

$$k'_n k_j \omega' \omega \frac{(2\pi)^4}{i}\,\delta(\Delta k + \Delta p) \sum_{\mathrm{N}} \left\{ \frac{\langle \mathrm{f}|\hat{d}_n|\mathrm{N}\rangle\langle \mathrm{N}|\hat{d}_j|0\rangle + \langle \mathrm{f}|\hat{d}_j|\mathrm{N}\rangle\langle \mathrm{N}|\hat{d}_n|0\rangle}{E_{\mathrm{N}} - E_0} \right\}$$

where the $\hat{d}_n$ are the operators of the components of the dipole moment $\int \hat{\varrho}(\mathbf{x})\mathbf{x}_n \mathrm{d}\mathbf{x}$.

In this eq. the states $|0\rangle$ and $|\mathrm{f}\rangle$ may differ only by the quantum number m of spin orientation. The quantity $\sum\{\ldots\}$ may be written as $\langle jm'|\hat{\alpha}_{nj}|jm\rangle$, where

$\hat{\alpha}_{nj} = \hat{\alpha}_{jn}$ corresponds to the usual definition of the tensor of electric dipole polarizability of the system † (nucleon in our case). For spin $\frac{1}{2}$,

$$\langle \tfrac{1}{2}m' | \hat{\alpha}_{nj} | \tfrac{1}{2}m \rangle = \langle \tfrac{1}{2} | \alpha | \tfrac{1}{2} \rangle \delta_{m'm} \delta_{nj}.$$

Now, we have the following expression for the average value of the electric polarizability of the nucleon:

$$\bar{\alpha} = \langle \tfrac{1}{2} | \alpha | \tfrac{1}{2} \rangle = 2 \sum_{N} \frac{|\langle N | \hat{d}_n | 0 \rangle|^2}{E_N - E_0}. \tag{1}$$

From eq. (1) it follows that all excited states of the system yield a positive contribution to the polarizability of the nucleon. The first state for which $\langle N | \hat{d}_n | 0 \rangle \neq 0$ is a meson+ nucleon state. The square of this matrix element may be expressed through the cross section for the electric dipole photoproduction of mesons σ_{E1}. Hence a lower estimate of $\bar{\alpha}$ may be found as

$$\bar{\alpha} > \frac{1}{2\pi^2} \int \frac{\sigma_{E1}}{\omega^2} \, d\omega. \tag{2}$$

Apart from that, eq. (1) may yield an estimate of the mean square fluctuation of the dipole moment in the ground state. Using the completeness of the system of functions we find

$$\bar{\alpha} = \tfrac{2}{3} \frac{1}{(E_N - E_0)} \langle 0 | \hat{d}^2 | 0 \rangle < \tfrac{2}{3} \frac{1}{\mu} \langle 0 | d^2 | 0 \rangle, \tag{3}$$

where μ is the mass of the meson; in the following we put $\mu = \hbar = c = 1$. If it turns out that the main contribution to nucleon polarizability comes from one-meson states the mean square fluctuation of the meson-nucleon distance may be estimated on the basis of eq. (3):

$$\frac{1}{e} \langle 0 | \int \hat{\varrho}(\mathbf{x}) \mathbf{x}_i d\mathbf{x} \cdot \int \hat{\varrho}(\mathbf{y}) \mathbf{y}_i d\mathbf{y} | 0 \rangle^{\frac{1}{2}} = \sqrt{\bar{r}^2} > \sqrt{\frac{3\bar{\alpha}}{2e^2}}, \tag{4}$$

where e is the charge. It should be mentioned that this quantity differs from $\langle 0 | \int \hat{\varrho}(\mathbf{x}) \mathbf{x}^2 d\mathbf{x} | 0 \rangle$, which is measured in electron-nucleon scattering experiments.

The exact value of $\bar{\alpha}$ may in principle be obtained from the data concerning the Compton effect on nucleon in the region of low frequencies. For this purpose the cross section, obtained by adding

$$-\bar{\alpha}\omega^2 \mathbf{e}' \cdot \mathbf{e} - \bar{\beta}(\mathbf{k}' \times \mathbf{e}') \cdot (\mathbf{k} \times \mathbf{e})$$

† If we define $\hat{g}_{nj} = \hat{g}_{nj}^{(e)} + \hat{g}_{nj}^{(M)}$, where $\hat{g}_{nj}^{(e)}$ describes the scattering of "electrical" photons only (E → E transitions) and $\hat{g}_{nj}^{(M)}$ all other types of scattering (M → M, E → M; and M → E transitions) then $k'_n \hat{g}_{nj} k_j = k_n \hat{g}_{nj}^{(e)} k$. The amplitude $\hat{\alpha}_{nj}$ is responsible for the Rayleigh scattering part of the $\hat{g}_{nj}^{(e)}$.

($\bar{\alpha}$ being the electric and $\bar{\beta}$ the magnetic polarizability of the nucleon) to the amplitude of ref. [1]), should be compared with experiment.

It is not difficult to show, for example, that the cross section for scattering at an angle of 90° (in the centre of mass system) may serve as a measure of the electric dipole polarizability:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{90^\circ} = \left(\frac{e^2}{M}\right)^2 \left[\frac{1}{2}\left(1-2\frac{\omega}{M}\right) + \omega^2\left(\mathscr{M} - \frac{\bar{\alpha}}{\left(\frac{e^2}{M}\right)}\right)\right]. \tag{5}$$

Here $\mathscr{M}$ is the part of the cross section determined by the static magnetic moment. Even the experimental data available at present enable us to make an upper estimate of the values of the quantities (3) and (4) on the basis of eq. (5). The region of applicability of such description calls for an additional study; this is due to the fact that the experiments on Compton effect are conducted only in the region of relatively high frequencies (in the range of 50 to 100 MeV). The upper limit of frequencies ω for which this description is correct within a given guaranteed accuracy (for example $\approx 5\,\%$) may be obtained through dispersion relations for the forward amplitude [2])

$$\begin{aligned} \operatorname{Re} f &= -\frac{e^2}{M} + \frac{\omega^2}{2\pi^2}\int_{\omega_0}^{\infty} \frac{\sigma(\omega')\mathrm{d}\omega'}{\omega'^2-\omega^2} \\ &\approx -\frac{e^2}{M} + \frac{\omega^2}{2\pi^2}\left\{\int_{\omega_0}^{\infty}\frac{\sigma(\omega')\mathrm{d}\omega'}{\omega'^2} + \omega^2\int_{\omega_0}^{\infty}\frac{\sigma(\omega')\mathrm{d}\omega'}{\omega'^4} + \dots\right\} \\ &= -\frac{e^2}{M} + \omega^2(\alpha+\beta) + \omega^2(\alpha+\beta)\left(\frac{\omega}{\bar{\omega}}\right)^2. \end{aligned} \tag{6}$$

Here $\sigma(\omega')$ is the total cross section for meson photoproduction. If it turns out that the region of large ω' contributes to the dispersion integrals the convergence of the series will be all the better. Even a rough estimate shows that the third term accounts for less than 2 % of the first one, up to photon energies in the range of 100 MeV. A number of estimates of the type of eq. (6) may be made using other disperision relations obtained on the basis of ref. [3]) and under the assumption that $\sigma(\omega')$ is determined only by low multipoles (but without using the experimental values of these). All estimates show that the contribution to the amplitude of the terms with powers of ω higher than the second does not exceed 0.03 for photon energies up to 100 MeV †. At the same time this result shows that the interaction of a nucleon and electromagnetic field can be described by four constants in a wide frequency range.

† The contribution from the difference $\alpha - \bar{\alpha} = \frac{1}{3}\frac{e}{M}\langle 0|\int \rho(\mathbf{x})\mathbf{x}^2\mathrm{d}\mathbf{x}|0\rangle$ between α (eq. (6)) and $\bar{\alpha}$ (eq. (1)) has the same small value.

It should be emphasized, however, that for the time being we have no test of this important proposition. The best test would be the measurement of the energy dependence of the cross section for Compton effect below the meson photoproduction threshold; i.e. it should be investigated, whether the energy dependence is of the kind

$$\frac{d\sigma}{d\Omega} = a+b\omega+c\omega^2.$$

3. Electric Polarizability of Protons

Substituting experimental values of the meson photoproduction cross-section into eqs. (2) and (6) we find

$$\bar{\alpha}_p > 0.4\times 10^{-42}\ \text{cm}^3, \quad \alpha_p+\beta_p \approx 1.07\times 10^{-42}\ \text{cm}^3$$

(neglecting the region of ω' higher than 1000 MeV in the dispersion integral).

Experimental data on the Compton effect [4]) show that the cross section $(d\sigma/d\Omega)_{90^\circ}$ is close to the Thomson cross section, whence the upper estimate for α_p follows:

$$\alpha_p \lesssim 1.5\times 10^{-42}\ \text{cm}^3.$$

Thus for the proton we have

$$0.4\times 10^{-42} \lesssim \bar{\alpha}_p \lesssim 1.5\times 10^{-42}\ \text{cm}^3. \tag{7}$$

In connection with this value it should be mentioned that an estimate of $\bar{\alpha}$ was recently made by V. S. Barashenkov and B. M. Barbashov [5]) on the basis of field theory with extended source; their result is $\bar{\alpha} = 1.8\times 10^{-42}$ cm^3. In refs. [6, 7]) estimates of $\bar{\alpha}$, obtained only on the basis of photoproduction data, are seen to coincide with our eq. (7). Recently, V. I. Goldansky *et al.* [8]) made an analysis of their new data on the Compton effect on protons at energies from 40 to 70 MeV. These authors have reduced our upper estimate to 0.9×10^{-42} cm^3.

The mean square fluctuations $\sqrt{\bar{r}^2}$ corresponding to the values of inequality (7) are 0.26×10^{-13} cm and 0.52×10^{-13} cm according to eq. (4). It is noteworthy that if we put $\overline{(E_N-E_0)} \approx 2\mu$ we obtain 0.7×10^{-13} cm on the basis of the upper estimate of α; this value is close to the "electric radius of a proton".

4. Polarizability of Neutrons

Only the lower limit for $\bar{\alpha}_n$ can be estimated on the basis of photoproduction data. These estimates differ little from those for $\bar{\alpha}_p$ according to eqs. (2) and (6). The quantity α_n should exceed α_p by something like 20 %, for the cross sections for photoproduction of neutron and proton are close to each other.

In ref. [9]) the problem was raised of a possible influence of the electric dipole polarizability of a neutron on the scattering of neutrons on heavy nuclei at small angles. The measurements of this effect yielded an estimate $\alpha_n \approx 8 \times 10^{-41}$ cm^3 which strongly contradicts eq. (7). Recently this question was again raised in ref. [6]). This paper discussed anisotropy in the scattering of slow neutrons (energy of the order of a few 100 keV) by nuclei. For a quantitative explanation of the anisotropy under study in this scattering it is necessary to assume $\alpha_n \approx 2 \times 10^{-41}$ cm^3, i.e. still one order of magnitude higher than our upper limit of eq. (7).

We might confine ourselves to mentioning this contradiction, regarding the interpretation of nuclear data as ambiguous. Yet we have no strict upper estimate of α_n at present since there are no data on the Compton effect on neutrons. If, on the other hand, we turn to the Compton effect on deuterium the new data of ref. [10]) indicate an abnormally large cross section for this process, namely $\sigma_d{}^{\gamma\gamma'} \approx 1.6\ \sigma_H{}^{\gamma\gamma'}$. The authors of ref. [10]) were unable to account for this value on the basis of the impulse approximation. In general the large value of the cross section σ_d thus observed might be accounted for by an abnormally large polarizability of the neutron, which follows from the data on the scattering of neutrons on nuclei. This conclusion seems to us premature, however, for both experiment and theory of the two effects need essentially greater accuracy.

Let us consider the Compton effect on deuterium in impulse approximation; in contrast to ref. [11]) we shall consistently take into account the effects of the interaction of the nucleons. In the transition amplitude we shall consider the contribution of intermediate states corresponding to the photodisintegration of the deuteron (a correction to the impulse approximation)

$$\hat{T} = \hat{t}^p + \hat{t}^n - i\pi \sum_N |\hat{V}_e|N\rangle\langle N|\hat{V}_e|. \tag{8}$$

Here, $\hat{t}^p$ and $\hat{t}^n$ are the scattering amplitudes of free nucleons having, as is well known, the shape

$$(\sigma \cdot \mathbf{K} + L)e^{i(\mathbf{k}'-\mathbf{k})\cdot \mathbf{r}};$$

$\hat{V}_e$ is the operator of interaction of electromagnetic field and a system of two nucleons. An accurate estimate of the third term in the amplitude is at present greatly dependent on model conceptions. Therefore, we shall merely make a rough estimate of this contribution for the simplest case. Let the interaction of the electromagnetic field with a deuteron reduce to the interaction with a proton charge and let the photodisintegration of the deuteron be of the electric dipole type. Let us also take into account that the state $|N\rangle$ describes the relative motion of two nucleons with small wavelengths. Under these assumptions we have

$$\hat{T} = \mathbf{e}\cdot\mathbf{e}' \frac{e^2}{M}\frac{2\pi}{\omega} e^{i(\mathbf{k}'-\mathbf{k})\cdot \mathbf{r}_p} - i\pi \frac{\sigma_d}{2\pi} \delta(\mathbf{r}_p - \mathbf{r}_n) \frac{\mathbf{e}'\cdot\mathbf{e}}{[\varphi_d(\mathbf{r}_p-\mathbf{r}_n)]^2} e^{\frac{1}{2}i(\mathbf{k}'-\mathbf{k})\cdot(\mathbf{r}_p+\mathbf{r}_n)}, \tag{9}$$

where σ_d is the cross section for the photodisintegration of the deuteron, φ_d is its wave function and $\mathbf{r}_p$ and $\mathbf{r}_n$ are the radii-vectors of the proton and the neutron. Calculating the cross section with $\hat{T}$ from eq. (9) and taking account of the neutron-proton interaction in the final state we find that the contribution of the interference term of the first and second parts of $\hat{T}$ are of the same order of magnitude (up to 70 %) as the cross section obtained after neglecting the second term. If no account is taken of the interaction in the final state, the interference term is equal to zero. This result shows that the interpretation of data on Compton effect on deuterium on the basis of the impulse approximation is not tenable. It should be mentioned, however, that the attempt at explaining the large value of the cross section $\sigma_d^{\gamma\gamma'}$ by the contribution of these interference terms is not successful. In integrating over the momentum ($\mathbf{p}$ of the relative motion of two nucleons the second term in eq. (9) yields a very low contribution. Yet this result does not show that the contribution of the interference terms to the cross section integrated over $\mathbf{p}$ will always be small.

In view of such uncertainty in the theory of the impulse approximation we think it rather desriable to measure the cross section of the reactions $\gamma+\mathrm{d} \rightarrow \mathrm{d}+\gamma'$ or $\gamma+\mathrm{H_e}^4 \rightarrow \mathrm{H_e}^4+\gamma'$ for which no such uncertainty with respect to interference terms exists and the contribution of the second part of amplitude (9) lies beyond the accuracy of all our calculations. It should also be noted that the contribution of the Low amplitude determined by the interaction of the pair of γ-quanta with a nucleon through the virtual π^0-meson equals zero for these reactions. This fact also eliminates a number of uncertainties in the interpretation of the data.

Let us represent the cross section of the reaction $\gamma+\mathrm{d} \rightarrow \mathrm{d}+\gamma'$ in a form which does not depend on the behaviour of the wave function in the region of nuclear forces:

$$\frac{\mathrm{d}\sigma_d^{\gamma\gamma'}}{\mathrm{d}\Omega} = \frac{k'^3}{(2\pi)^2\left[k' + \dfrac{k'^2-(\mathbf{k}'\cdot\mathbf{k})}{2M}\right]} \cdot \left\{\tfrac{2}{3}|\mathbf{K}_p+\mathbf{K}_n|^2+|L_p+L_n|^2\right\}\left(\frac{\gamma}{1-\gamma\rho_t}\left[\frac{2}{q}\,\mathrm{arctg}\,\frac{q}{2\gamma}-\rho_t\right]\right)^2 \tag{10}$$

Here $\gamma = \sqrt{M\varepsilon}$ where ε is the binding energy of the deuteron and ρ_t is the triplet effective radius, $q = \sqrt{k'^2+k^2-2(\mathbf{k}'\cdot\mathbf{k})}$. Eq. (10) is very critical with respect to the polarizability of the neutron, for the first term within brackets is small ($\mathbf{K}_p$, $\mathbf{K}_n$ have opposite signs), while the signs of the amplitudes with polarizabilities in L_p and L_n are opposite to the sign of the Thomson scattering amplitude.

A large value of α_n seems improbable because of the smallness of the interval between the upper and lower estimates of eq. (7). Yet this probability should be borne in mind, especially in connection with the data on electron-neutron scattering which have revealed an essential difference in the electromagnetic characteristics of the neutron and the proton.

References

1) F. Low, Phys. Rev. **96** (1954) 1428;
M. Gell-Mann and H. Z. Goldberger, Phys. Rev. **96** (1954) 1433;
A. Klein, Phys. Rev. **99** (1955) 998
2) M. Gell-Mann, M. L. Goldberg and W. E. Thirring, Phys. Rev. **95** (1954) 1612
3) N. N. Bogolyubov and V. D. Shirkov, DAN SSSR **113** (1957) 529
4) B. B. Govorkov, V. I. Goldansky, O. A. Karpukhin, A. V. Kutsenko and V. V. Pavlovskaya, DAN SSSR **111** (1956) 988;
G. E. Pugh, R. Gomez, D. H. Frish and G. S. Janes, Phys. Rev. **105** (1957) 982
5) V. S. Barashenkov and B. M. Barbashov (preprint of the Joint Institute for Nuclear Research, Dubna); see also L. Schiff (report at the Kiev Conference, July 1959)
6) R. M. Thaler, Phys. Rev. **114** (1959) 827
7). G. Breit and M. Z. Rustgi, Phys. Rev. **114** (1959) 830
8) V. I. Goldansky, O. A. Karpukhin, A. V. Kutsenko and V. V. Pavlovskaya (see report G. Bernardini at the Kiev Conference July, 1959)
9) Yu. A. Alexandrov and I. I. Bondarenko, JETP **31** (1956) 726;
B. S. Barashenkov, I. P. Stakhanov and Yu. A. Alexandrov, JETP **32** (1957) 154
10) L. G. Hyman, R. Ely, D. H. Frisch and M. A. Wahlig, Phys. Rev. Letters **3** (1959) 93
11) R. H. Capps, Phys. Rev. **106** (1956) 1031
12) C. Oxley, V. Telegdi P. Rev 100 1435 (1955)
13) G. Pugh, R Gomez D Frisch, G Janes Phys Rev 100 1245 (1955)