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Multibaryon Interactions at Relativistic Energies

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In the past years there has been a great growth of interest in studying the collisions of relativistic nuclei and particles with nuclei. These studies are essentially beyond the framework of the canonical nonrelativistic nuclear theory, according to which the nucleus is thought of as a system of nucleons described by the Schroedinger equation. Large momentum transfers to a nucleon system (of the order of or larger than the nucleon mass) require a consistent relativistic approach and correspond to so small relative internucleon distances that not only the concept of potential, but also the concept of nucleon itself as a quasiparticle adequate to the problem lose their sense. For the relative internucleon distances of the order of or smaller than the confinement radius, the quark degrees of freedom must play an important role. As a matter of fact, we are dealing here with the problems of the hadron physics and quantum field theory. Correspondingly, the methods and approaches in these both theoretical and experimental investigations are essentially an adaptation and development of the high-energy physics methods.

The problem of description of composite systems for which the relativistic effects are essential has recently been of paramount importance due to the quark models of hadrons. As far as true elementary objects are presently taken to be quarks and it may be suggested a far-reaching analogy of the particles with the atoms and molecules, then the processes of relativistic nuclear physics seem to be analogs of chemical reactions and processes of plasma physics.

The investigation of the elementary particle structure at short distances needs study of large momentum transfers when the wave length of interacting objects is much smaller than their sizes. The large momentum transfers to particles result inevitably in the production of a large number of new particles. In the study of elementary particle interactions at very high energies we are dealing with systems possessing an infinte number of degrees of freedom. In this sense the transition to the interaction of nuclei adds only a finite number of degrees of freedom to the infinite one. The nuclear collisions in the relativistic region are not found to be more complicated that the hadron collisions. Moreover, the collision properties of relativistic nuclei are found to be similar to those of protons at high energies. Experimental methods for studying these processes have been developed and have successfully been applied in the physics of relativistic nuclei. In hadron physics, the study of multiple production and deep inelastic processes has enabled one to formulate very important notions such as scale invariance and asymptotic freedom of quarks, to check the basic model ideas about the hadron structure.

The question naturally arises as to what new information we obtain by studying large momentum transfers to nuclei. Sceptics asserted, and similar assertions still occur at present, that the investigations of interactions of relativistic nuclei would be a duplication of the investigations of nucleon properties but would contain some unnecessary complications due to the Fermi motion of nucleons and unknown properties the nucleus wave function at small distances. In the present lectures we try to show the invalidity of such assertions and describe the experimental studies, which have resulted in the discovery of nontrivial multibaryon phenomena and the theoretical studies which have demonstrated the relationship between these phenomena and the main problems of construction of the quark theory of matter. The collisions of nuclei in the relativistic region were found to be even more informative than the collisions of particles. This is due to the fact that we know more about the nucleus structure than the proton structure and can strongly vary the parameters of colliding objects. We can check the ideas of space-time picture of hadron production and obtain additional information which can be extracted by no other way and which is very sensitive to strong interaction models under development.

The interactions of nuclei in the relativistic energy region are at present a wide and fast developing field of high-energy physics which is also of great importance for understanding the properties of atomic nuclei. In the framework of the present lectures, I have no possibility of discussing in detail this domain and I will dwell on the problems which seem to me to be especilaly important. The lectures are intended for nuclear physicists who are interested in the development of nuclear physics in the new energy region. The first section is thus devoted to the introduction of some important concepts which are not yet generally accepted in nuclear physics and are related to the particle physics.

The processes of relativistic nuclear physics studied by means of modern methods of high-energy physics give very abundant information. Unfortunately, the overwhelming part of the data is not sensitive to the existing models. Many characteristics are of a purely statistical character and are described by the Poisson distributions, or reflect really the nucleus properties defined by large distances and the Fermi motion. In this connection, it is very important to formulate the problem in an optimal manner, to find the conditions of "informative comfort". It seem to us that the formulations of the problems associated in some way or other with large momentum transfers to the nucleon system are most important.

In the present lectures we pay a special attention to the cumulative effect. By cumulative particles we mean * the particles produced on nuclei in the region kinematically forbidden for one-nucleon collisions. Extraction of the events containing cumulative particles identifies the configurations in the nucleus wave function when several nucleons are at distances smaller than the sizes of one nucleon. At such short distances the quark-parton constituents of nucleons appear to be collectivized which, according to the predictions, must result in the formation of unusual objects of hadron physics. A large amount of appropriate experimental information obtained at Dubna, which will be reviewed below, has attracted a considerable attention of theoretists. I will describe the known to me attempts of interpreting experimentally discovered phenomena on the basis of the methods of theoretical physics.

The multibaryon configurations which are responsible for the cumulative effect are also of great interest because of possible existence of metastable multiquark systems, for example, of dibaryons. The application of the quark bag theory to these states show that they can be interpreted as a discovery of super-dense states of nuclear matter. Rather interesting theoretical and experimental studies are being performed along these lines at Dubna, I will give a brief survey of them.

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* A strict definition of the cumulative effect is given in Section II.

In 1970 the systems of the Dubna Synchrophasotron were adjusted to accelerate relativistic nuclei. Since that time a wide program of investigations in the field of relativistic nuclear physics has been realizing at the Joint Institute for Nuclear Research. A sketchy description of this program is also the subject of the present lectures.

To conclude I will discuss some perspectives of the development of an accelerator complex which is planned to be constructed on the basis of the Dubna Synchrophasotron.

I. The Basic Notions and Quantities

The region of multibaryon phenomena defined by the condition

$$\frac{\vec{p}^{\,2}}{m^2} \gg 1,$$

where $\vec{p}$ are the three-momenta of particles and m their masses, is called by us the relativistic nuclear physics. The production of new particles is very essential in this region. The number of the degrees of freedom, the number of the variables, which describes the result of hadron collisions in this region is so large that from the very beginning we are forced to use an incomplete description. The requirements of the theory of relativity are very important and the mathematic apparatus formulated on the basis of relativistic invariant quantities, for example, squared four vectors, is found to be most suitable.

In hadron physics, in the analysis of experimental data, the quantities to be measured are invariant inclusive cross sections (one-, two- and so on particle distributions):

$$\rho_1 = \frac{E}{\sigma_{in}} \frac{d\sigma}{d\vec{p}}; \qquad \rho_2 = \frac{E_1 E_2}{\sigma_{in}} \cdot \frac{d\sigma}{d\vec{p}_1\, d\vec{p}_2}, \text{etc}. \tag{1.2}$$

which correspond to the identification in the final state of one, two and so on particles:

$$I + II \to 1 + X$$
$$I + II \to 1 + 2 + X. \tag{1.3}$$

The quantities ρ_1, ρ_2 and so on depend on the relativistic invariants, in particular, on $s = (p_I + p_{II})^2 = m_I^2 + m_{II}^2 + 2(p_I \cdot p_{II})$; σ_{in} is the inelastic cross section for reactions induced by the collision of systems I and II ; p_I and p_{II} are the four-momenta of particles I and II . The ρ_m is determined by the following manner. Let n particles of the type a $(n > m)$ be produced in a collision

$$I + II \to i_1 + i_2 + \ldots + i_n + X$$

where x is a system of hadrons containing no particles of the type a . We denote the cross section of such a process as $d\sigma(n, X)$.

We integrate $d\sigma(n, X)$ over the phase volume of system X and take the sum over the all possible channels of X . Then we obtain the production cross section for n particles of the type a with an arbitrary hadronic accompaniment $d\sigma(n)$

$$d\sigma(n) = f(p_1, \ldots p_n, p_I, p_{II}) \frac{1}{n!} \prod_{j=1}^{n} \frac{d\vec{p}_j}{E_j} \tag{1.4}$$

f is a relativistic invariant.

We integrate eq..(1.4) over the all momenta of the particles of the type a , for the exception of m , and divide by σ_{in} :

$$\rho_m = \rho(p_1, \ldots, p_m, p_I, p_{II}) = \frac{1}{\sigma_{in}} E_1 E_2 \ldots E_m \frac{d\sigma}{d\vec{p}_1 \ldots d\vec{p}_m}$$

In virtue of the identity of the particles of the type a, ρ_m is normalized as follows

$$\int \rho_m(p_1,\dots p_m,p_I p_{II})\frac{d\vec{p}_1}{E_1}\dots\frac{d\vec{p}_m}{E_m} = \langle n(n-1)\dots(n-m+1)\rangle. \qquad (1.5)$$

The symbol < > denotes an averaging over all the observed events with $n > m$ outgoing particles.

In terms of the discrete probability $P_n = \frac{\sigma_n}{\sigma_{in}}$ the r.h.s. of eq. (1.5) can also be expressed as follows:

$$\langle n(n-1)\dots(n-m+1)\rangle = \sum_{n=m}^{\infty} n(n-1)\dots(n-m+1)P_n \quad . \qquad (1.6)$$

The summation can start with $n = 0$, since at $n < m$ the factor for P_n vanishes.

Thus, the basic quantities to be obtained in experiments and discussed by theoretists are the invariant quantities which depend on s,the four-momenta and,generally speaking,on other quantum numbers of the particles involved in the reaction. Instead of the relativistic invariants of the type $s = (p_I + p_{II})^2$; $t=(p_I - p_1)^2$ and so on, as the ρ arguments use is often made of the quantities which are called rapidities. The important properties of ρ , which will be essential for the following presentation, are simply expressed in terms of these quantities. The introduction of the rapidities is also necessitated by the difference between the longitudinal (along the collision axis) and transverse motions. By the rapidity we mean the quantity

$$y = \frac{1}{2}\ln\frac{E + p_z}{E - p_z} = \text{arsh}\,\frac{p_z}{\mu} \qquad (1.7)$$

here and in what follows $\mu_i = \sqrt{p_{i\perp}^2 + m_i^2}$.

At nonrelativistic energies the rapidity reduces to a projection of the particle velocity on the axis z which is assumed to be the direction of collision of particles I and II : $y = v_z$.

We notice the remarkable property of the rapidity for the Lorentz transformations along the axis z . It transforms additively. Respectively, the rapidity difference remains invariant. In fact, we find $y' = \ln\frac{E' + p'_z}{\mu}$. With the Lorentz transformation

$$E' = \frac{E + v p_z}{\sqrt{1 - v^2}}\ ; \quad p'_z = \frac{p_z + vE}{\sqrt{1 - v^2}} \quad \text{we get} \quad y' = y + \frac{1}{2}\ln\frac{1+v}{1-v} \ .$$

The connection of the i and j particle rapidity difference with the p_i and p_j four-vector product has the form

$$(p_i \cdot p_j) = \mu_i\mu_j \text{ch}(y_i - y_j) - \vec{p}_{i\perp}\cdot\vec{p}_{j\perp} \qquad (1.8)$$

In virtue of relativistic invariance, the distributions ρ_m depend on the rapidity differences. The dependence on the transverse momenta $p_\perp$ is for the moment omitted

$$\rho_1(y_I - y_1, y_1 - y_{II}) = \frac{1}{\sigma_{in}}\cdot\frac{d\sigma}{dy_1\, d^2p_{1\perp}}$$

$$\rho_2(y_I - y_1, y_1 - y_2, y_2 - y_{II}) = \frac{1}{\sigma_{in}}\cdot\frac{d\sigma}{dy_1 dy_2\, d^2p_{1\perp}\, d^2p_{2\perp}} . \qquad (1.9)$$

The invariant phase volume:

$$\frac{d\,\vec{p}_i}{E_i} = dy_i\, d^2 p_{i\perp}$$

For definiteness we assume the rapidities to be ordered

$$y_{II} \le y_2 < y_I \le y_1 \tag{1.10}$$

The important property of the ρ distributions, the so-called short-range order (SRO) model, was established as an approximate law of the high-energy hadron physics, which is formulated as follows.

1. If the rapidity difference is far larger than the characteristic correlation length L , then ρ_k is independent of this difference. In particular, ρ_k is independent of $(y_I - y_{II})$, if $|y_I - y_{II}| > L$.

2. If $y_I - y_2 > L$, then ρ is factorized to a product of one-particle distributions

$$\rho_2(y_1 - y_I, y_I - y_2, y_2 - y_{II})\Big|_{|y_I - y_2| > L} \to \rho_1(y_1 - y_I)\cdot\rho_1(y_2 - y_{II}). \tag{1.11}$$

These conditions are in a trivial manner generalized to any ρ_m, where $m \ge 3$. The case $|y_I - y_{II}| > L$ corresponds to a limiting fragmentation. The hypothesis of limiting fragmentation states that the limit:

$$\lim_{s\to\infty} \rho(s, y, p_\perp) = \rho_\infty(y_1, p_\perp)$$

exists. It will be clear from the following that the condition (1.1) provides an approximate limiting fragmentation. The limit $s \to \infty$ is defined by the magnitude of the correlation length L . In order to study the two-particle correlations in multiple particle production a correlation function

$$C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_2(y_2) \tag{1.12}$$

is defined. According to SRO, for $|y_1 - y_2| > L$, $C_2 = 0$, and respectively, ρ_2 is factorized. In hadron physics it is usually assumed that for large $|y_1 - y_2|$ values:

$$C_2(y_1, y_2) \propto \exp\left[-\frac{|y_1 - y_2|}{L}\right] \text{ and } L \approx 2 .$$

A priori there is no special reason to believe that there exists an universal correlation length L which is valid for all types of high-energy reactions. Moreover, strictly speaking, the correlations should not be only of short-range order, at least, because of the restrictions imposed by energy-momentum conservation. Nevertheless, the SRO model well describes many characteristics of multiple particle production and may be viewed as an approximate universal property of hadron interactions.

Nuclear collisions should obey the laws discovered in hadron physics and, in particular, the use of SRO is found to be especially efficient in relativistic nuclear physics.

Firstly, this model enables us to predict the region of approximate validity of limiting fragmentation. Recall

$$(p_I \cdot p_{II}) = m_I m_{II} \mathrm{ch}(y_I - y_{II}) \approx m_I m_{II} \frac{1}{2} \exp|y_I - y_{II}| . \tag{1.13}$$

The case $|y_I - y_{II}| \gtrsim 2$ corresponds to this approximate boundary, or

$$(p_I \cdot p_{II}) = E_I\, m_{II} \approx m_I m_{II} \frac{1}{2} \exp|y_I - y_{II}| \gtrsim m_I m_{II} \frac{1}{2} \exp 2 \tag{1.14}$$

or: $2\frac{E_I}{m_I} \gtrsim e^2 \approx 7.4$, i.e. at an energy E_I of about 4 GeV/nucleon. This boundary appears to be in agreement with the condition (1.1): $\frac{p_I^2}{m_I^2} \approx 14 \gg 1$. It will be still discussed in the lectures. Secondly, since for $E_I > 4$ GeV/N the happening near the left boundary in the rapidity space (in the vicinity of y_{II}) do not affect that in the vicinity of the right boundary (y_I), then the properties of limiting fragmentation of nucleus I must be independent of the properties of nucleus II. Hence it follows, in particular, that in order to study the limiting fragmentation of nuclei it is unnecessary to accelerate them. It is sufficient to study the particle production to a backward hemisphere under the action of any kind high-energy particles, This assertion is explained on fig.1, where a rapidity distribution is presented.

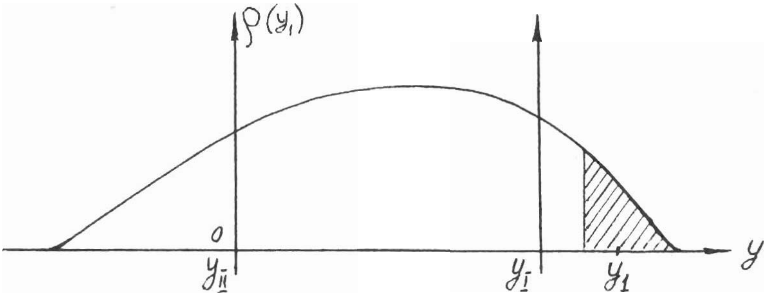

Fig.1

Let us assume that we are interested in a shaded area of the rapidity distribution, the region of limiting fragmentation of nucleus y_I, and that $|y_I - y_{II}| > 2$. If we choose a coordinate system where nucleus II is at rest, then y_I and y_1 are both positive, y_1 being larger than y_I. This corresponds to the case when the energy of particle 1 is higher than the energy per nucleon of fragmenting nucleus I.

If now we consider the same situation in the rest system of nucleus I then, as is seen from fig.1, the rapidities of nucleus II and particle 1 have opposite signs This corresponds to the fact that particle 1 moves in the direction opposite to the direction of motion of nucleus II (backward hemisphere). According to the limiting fragmentation property, in our case ρ is independent of $|y_{II} - y_I|$ and the properties of particle II since particle 1 is located in the rapidity space far from particle II. It is profitable to choose as particle II a light particle, e.g. a pion, instead of a nucleus. In so doing, the condition (1.14) is written as (particle II is moving)

$$2\frac{E_{II}}{m_{II}} \gtrsim 8$$

Thus, in the production of particles on nuclei under the action of pions, the limiting fragmentation is expected to begin at pion energies lower than 1 GeV. The same is valid for the particle production on nuclei to the backward hemisphere under the action of γ quanta and neutrinos, which will be discussed below.

The Lorentz transformation of the measured ρ to a system in which nucleus I is moving makes it possible to predict secondary beams which will be obtained, e.g., from acceleration of uranium.

It was just in this formulation that the properties of the limiting fragmentation of nuclei have been studying at Dubna since 1971.

The limits of change of y_1 are defined by energy-momentum conservation which can be written in the following form

$$(p_I + p_{II} - p_1)^2 = M^2 , \qquad (1.15)$$

where M^2 is the square of the missing mass.
We write eq.(1.15) in terms of the rapidities by denoting

$$a = y_I - y_{II} \; ; \; b = y_1 - y_I \; ; \quad y_1 - y_{II} = b + a \; .$$

We consider the case $y_1 > y_I > y_{II}$, $\exp a \gg 1$. Then, taking into account that $(p_I \cdot p_1) = m_I \mu \,\mathrm{ch}(y_1 - y_I)$:

$$m_I m_{II} \exp a - m_I \mu_1 [\exp b + \exp(-b)] - m_{II} \mu \exp(a+b) = \Delta , \tag{1.16}$$

where $\Delta = M^2 - m_I^2 - m_{II}^2 - m_1^2$

or

$$m_I - \frac{m_I \cdot \mu}{m_{II}} \cdot \frac{\exp b + \exp(-b)}{\exp a} - \mu \exp b = \frac{\Delta}{m_{II} \exp a} \tag{1.17}$$

This case corresponds to the limiting fragmentation of nucleus I and the largest possible value of $b = y_1 - y_{II}$ (the edge of the shaded area in fig.1) is mainly determined by the mass m_I involved in the collision.

Our conclusion that the collisions of pions, and even γ quanta and neutrinos, with uranium are equivalent to the uranium-uranium collisions seems at fisrt glance to be rather paradoxical even if the small area of the rapidity space (shaded in our figure) is concerned. The solution to this paradox consists in that the interaction of hadrons and, in particular, of nuclei in the region of large momentum transfers has a local character.

It was just this hypothesis that we used from the very beginning as a basis for considering the collisions of relativistic nuclei.

The local charcater of hadron interaction was naturally associated with the experimentally discovered property of scale invariance or automodelity of strong interactions. We explain these properties. The scale invariance is contained in SRO. In fact, the dependence of ρ on $(y_I - y_1)$ can be expressed by means of eq.(1.8) as a function of the ratio of the four-momentum products

$$\frac{(p_1 \cdot p_{II})}{(p_I \cdot p_{II})} \approx \frac{\mu}{m_I} \exp(y_1 - y_I) \tag{1.18}$$

The dependence of the cross sections on only the ratio of momenta and the independence of s can be obtained by requiring invariance with respect to the transformations of all the momenta of the type

$$p_i \to \lambda p_i , \tag{1.19}$$

where λ is constant. As far as (1.19) corresponds to the scale transformation in the momentum and coordinate space, then the appropriate property was named scale invariance or scaling. A similar property is realized in macroscopic physics in those cases when the problem contains no parameters of dimention of length. In our case the condition (1.1) garantees the possibility of neglecting the masses which in our units $(h = c = 1)$, have dimension cm^{-1}. In addition, the condition (1.1) implies so small particle wave lengths that the form factors, internucleon distances in nuclei and other parameters of dimention of length become nonessential. The notion of automodelity is taken from the point-like explosion theory[2)] in which the detonation wave is considered in an approximation when the dimensions of the region of the explosion can be neglected. In this case many wave properties can be obtained from dimensional analysis. The dimensional analysis of the hadron interaction cross sections at high energies based on the neglect of the involved particle masses was found to be very useful[3)].

It is natural that scale invariance should not so much be postulated as deduced from a specific dynamics of interactions. There exists a large number of papers in which both scale invariance and its limitedness not only on the side of low, but also on the side of high energies are derived from quantum field theory. So, we want to stress that both SRO and scale invariance are approximate properties of hadronic matter and need further be studied.

Our application of the rapidity space to nuclear physics needs also be explained as to how the description of ρ in this space is affected by the relative smallness of the nucleon binding energy. The nucleus is a relatively weakly bound system, and the stripping and pickup reactions must obviously be of much importance in nuclear collisions at any energies. The smallness of the energy parameter (mass, binding, energy and so on) in its most general form is exhibited in the existence of singularities (more often poles) in an unphysical domain, very close to its boundary. This is just a very simple method of parametrization of the problem in the general form which does not require any interaction mechanism models.

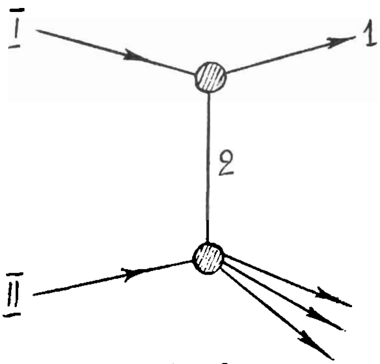

Fig. 2

We apply the pole diagram of fig.2 to the nucleus collision reaction $I + II \to 1 + \dots$ where the line 2 stands for exchange of a nucleus in the ground state. Then the amplitude of the process has the form:

$$T_{fi} = \frac{1}{2} \sum_j \frac{T_{fj} T_{ji}}{(p_I - p_1)^2 - m_2^2}. \qquad (1.20)$$

The existence of this pole singularity follows from the basic principles of quantum mechanics such as analyticity and unitarity of the amplitude and requires no other assumptions. We transform the denominator as follows:

$$(p_I - p_1)^2 - m_2^2 = m_I^2 + m_1^2 - 2(p_I \cdot p_1) - m_2^2 + 2m_I m_1 - 2m_I m_1 =$$
$$= -(m_1 + m_2 - m_I)(m_I + m_2 - m_1) - 2[(p_I \cdot p_1) - m_I m_1] \qquad (1.21)$$

or introducing

$$b_{I1} = 2\left[\frac{(p_I \cdot p_1)}{m_I m_1} - 1\right] \quad \text{and} \quad \alpha = \frac{(m_1 + m_2 - m_I)(m_I + m_2 - m_1)}{m_I m_1}$$

we write the process cross section in terms of the squared matrix element (1.20) [4]:

$$\frac{d\sigma}{db_{I1}} = \frac{F}{(\alpha + b_{I1})^2} \qquad (1.22)$$

The quantity

$$\alpha \approx \frac{2|\epsilon|\,|m_I - m_1|}{m_I m_1} \qquad (1.23)$$

It is equal in the order of magnitude to the ratio of the binding energy to the nuclear fragment mass or $\alpha \sim 10^{-2}$. The fraction with a denominator such as in eq. (1.22) behaves as a δ function

$|\epsilon| = |m_I - m_1 - m_2|$ stripping reaction

$|\epsilon| = |m_1 - m_I - m_2|$ pick-up reaction.

Now we consider b_{I1}. In the rest system of nucleus I b_{I1} is expressed as

$$b_{I1} = 2\left[\frac{(p_I \cdot p_1)}{m_I m_1} - 1\right] = 2\left[\left(1 + \frac{\vec{p}_1^{\,2}}{m_1^2}\right)^{1/2} - 1\right] \approx \frac{\vec{p}_1^{\,2}}{m_1^2} = \frac{p_{1z}^2 + p_{1\perp}^2}{m_1^2} \qquad (1.24)$$

and in terms of the rapidities y_I and y_1 as:

$$2\left[\frac{\mu_1}{m_1}\,\mathrm{ch}\,(y_I - y_1) - 1\right] \approx (y_I - y_1)^2 \qquad (1.25)$$

Comparing (1.22),(1.23) and (1.25) we see that the rapidity distribution width is

$$\Delta y \approx \sqrt{2|\epsilon|\,\frac{|m_I - m_1|}{m_I \cdot m_{II}}} \approx 0.1 .$$

Thus, the smallness of the binding energy will be revealed in the rapidity space throught super-short-range correlations. It is seen from eq. (1.24) that these effects belong to the low-energy nuclear physics when the kinetic energy of the reaction products is in its magnitude close to the binding energy (Fermi motion). An explicit Lorentz invariant form of the cross section makes it possible to consider the super-short-range correlations of relativistic nuclei on the basis of eqs. (1.22) and (1.25) in any coordinate system. The cross sections of these processes are very large due to the small denominator $F/\alpha^2 \sim 10^4 F$. They are a large part of the total collision cross section of the relativistic nucleus but contain information about only its low-energy properties (large distances). This process is referred to as nuclear fragment production. A detailed study of this process was performed in Berkeley[5]. The main laws of nuclear fragmentation indicated by the authors of the cited paper are the following:

i) The fragmentation cross sections are factorized $\sigma_{II}^{I} = C_I \cdot C_{II}$, i.e. each factor depends only on the properties of nuclei I and II.

ii) The average fragment velocities are equal to those of bombarding nuclei.

iii) The fragment momentum distributions are identical in the rest system of fragmenting nuclei. The longitudinal momentum distributions coincide with the transverse momentum distributions and can be described by a Gaussian

$$N = a \exp[-p^2/2\delta^2] , \text{ in this case } \delta \approx m_\pi \approx 140 \text{ MeV}.$$

The latter fact is stressed by the authors of ref.[5]

As is shown in ref.[4], these laws are easily explained by means of eq.(1.22). The introduction of the parameter b_{I1} alone, instead of longitudinal and transverse momenta, makes the analysis simpler.

The following conclusions are immediately drawn from eqs. (1.22), (1.23) and (1.24):

1. The cross section factorization is fulfilled in accordance with what one observed in experiment[5] and in order to explain it there is no need to refer to the Regge models: when the spin effects are neglected the quantity F of eqs. (1.22) contains the product of the squares $|T_{fj}|^2 \cdot |T_{ji}|^2$.

2. The equality of the average fragment velocities (rapidities) is a natural consequence of the validity of pole approximation. This is seen from eq. (1.22) where the denominator behaves like a δ function of the rapidity difference.

3. The momentum distribution has sharp peaks at

$$\frac{\mu_1}{p_{1z}} = \frac{m_I}{p_I} \quad \text{or at} \quad p_{1z} = m_1 \frac{p_I}{m_I} = m_1\,\text{const}$$

according to experiment. This follows from the fact that

$$b_{I1} = (p_{1z}\frac{p_I}{m_I})(\frac{m_I}{p_I} - \frac{\mu_1}{p_1}) + (\mu_1 - m_1)$$

4. The longitudinal and transverse momentum distributions coincide in the rest system of a fragmenting nucleus since b_{I1} in eq. (1.22) is equal to

$$(p_{1z}^2 + r_1^2)\frac{1}{m_1^2}$$

The assertion of the authors of ref.[5] that all the peaks in the fragment distribution are described by a unified Gaussian with $\delta = m_\pi = 140$ MeV is a consequence of an insufficient accuracy of experiment. The coincidence of the width with m_π is of a random character. According to eq.(1.22), the distribution width is given by eq.(1.23) which, in notation $m_1 = F$; $m_I = B$, takes the form

$$\alpha m_1^2 = \epsilon\,\frac{(B-F)F}{B}$$

in which it is also utilized under the name "parabolic law" (see also the lectures by H.Feschbach).

The exact formula should contain a summation over all the poles in the unphysical domain. The single-pole formula (1.22) well describes the nuclear fragmentation of light nuclei for which the levels of nucleus 2 are far from one another and, accordingly, we may restrict ourselves to the main pole alone. The nuclear fragmentation of heavy nuclei is described by a certain set of levels or by an effective pole defined by eq. (1.22) with ϵ as a fit parameter. The experimental determination of ϵ as a fit parameter and its comparison with $|m_I - m_1 + m_2|$, where m_2 is the mass of the ground state of nucleus 2, may serve as a criterium of validity of the single-pole approximation. Eq. (1.22) describes also the fragment angular distribution in a system where I moves with a relativistic velocity

$$b_{I1} = 2\left[\frac{E_I E_1 - p_I p_1 \cos\theta}{m_I m_1} - 1\right] \simeq 2\left[\frac{p_I}{m_I}\cdot\frac{p_1}{m_1}(1-\cos\theta)\right] \approx \frac{p_I^2}{m_I^2}\theta^2.$$

Hence the angular distribution of nuclear fragments is given by the formula

$$\frac{d\sigma}{(\frac{p_I}{m_I})^2 d\theta^2} = \frac{F}{[\alpha + \frac{p_I^2}{m_I^2}\theta^2]^2} \rightarrow \frac{d\sigma}{d\theta^2} \approx \frac{F\frac{m_I^2}{p_I^2}}{[\theta^2 + \alpha\frac{m_I^2}{p_I^2}]^2}$$

To conclude we give for convenience a summary of the basic quantities which are used in relativistic nuclear physics.

One-, two- and so on particle distributions

$$\rho_1 = \frac{E_1}{\sigma_{in}}\frac{d\sigma}{d\vec{p}_1}; \qquad \rho_2 = \frac{E_1 E_2}{\sigma_{in}}\cdot\frac{d\sigma}{d\vec{p}_1\, d\vec{p}_2}$$

Because of the fact that it is difficult to determine σ_{in} in relativistic nuclear physics, experimentalists prefer to present their data in the form

$$E_1\frac{d\sigma}{d\vec{p}_1} = \sigma_{in}\rho = f(s, y_1, r_1)$$

where $y_1 = \frac{1}{2}\ln\frac{E_1 + p_{1z}}{E_1 - p_{1z}}$ is the rapidity, $\mu_i = \sqrt{r_i^2 + m_i^2}$, $\vec{r}_i = \vec{p}_{i\perp}$ is the transverse momentum,

$$\frac{d\vec{p}_1}{E_1} = dy_1 d^2 r_1 \qquad \text{the phase volume}$$

The axis z coincides with the axis of collision of particles I and II in the multiple particle production reaction $I + II \to 1 + X$. The four-momentum products

$$(p_i \cdot p_j) = \mu_i \mu_j \, ch(y_i - y_j) - \vec{r}_i \vec{r}_j$$

$$s = (p_I + p_{II})^2 = m_I^2 + m_{II}^2 + 2 m_I m_{II} ch(y_I - y_{II}) \approx m_I m_{II} \exp|y_I - y_{II}|$$
$$\text{at } \exp|y_1 - y_I| \gg 1$$

Instead of y_1 , use is also made of a relativistically invariant quantity $N^{min} = \frac{\mu_1}{m_p} \exp|y_1 - y_I|$, the cumulative number (see below), and $T_1 = E_1 - m_1$, the kinetic energy of particles in the rest system of fragmenting nucleus I .

The limiting fragmentation

$$\lim \rho = \rho_\infty$$
$$\text{at } \exp|y_I - y_{II}| \to \infty$$

The kinematic boundary on y_1 for $y_1 \cdot y_I$ and $\exp|y_I - y_{II}| \gg 1$ (see. eq. (1.17) is defined by energy momentum conservation

$$\frac{\mu_1 \exp|y_1 - y_I|}{m_I} < 1 - \frac{M_X^2}{s} < 1 .$$

In the frame, where particle II is at rest, $\frac{\mu_1 \exp|y_1 - y_I|}{m_I} = \frac{E_1 + p_1}{E_I + p_I}$ is the momentum fraction of system I carried away by particle 1 .

II. The Cumulative Effect

In this section we focuss our attention on the particle production in the region of limiting fragmentation of nuclei which is kinematically forbidden for one-nucleon collisions (the cumulative region). The particle production in this region posses-ses a number of particular features and allows an obvious interpretation in terms of quark-parton models. This enables us to speak about a specific effect which we call the cumulative effect.

The first data on cumulative production of mesons by relativistic deuterons on nuclei were reported as early as in 1971 at the meeting of the American Physical Society[6)] and in 1972 at the XVI International Conference on High Energy Physics in Batavia[7)]. Since that time a great deal of information on this effect has been accumulated. In our studies the kinematic limits are defined by the cumulative number N , that is by the effective number of the nucleons of a fragmenting nuc-leus which are involved in the reaction. For the one-particle distributions the minimal value of N is determined by the kinematic limits imposed on the mass of the object taking part in the collisions

$$I + II \to 1 + X$$

When $\exp|y_I - y_{II}| \gg 1$ in the region of limiting fragmentation of nucleus I (we consider as usual the case $y_1 > y_I$) the relativistic invariant quantity N^{min} as-sumes, according to eq. (1.17), the following values

$$N^{min} = \frac{\mu_1 \exp|y_1 - y_I|}{m_p} = \begin{cases} \dfrac{E_1 - p_{1z}}{m_p} & \text{in the rest system of nucleus I} \\ \dfrac{E_1 + p_{1z}}{E_{0I} + p_I^\circ} \approx \dfrac{p_{1z}}{p_I^\circ} & \text{in the rest system of particle or nucleus II} \end{cases}$$

where p_I° is the momentum per nucleon, m_p the proton mass. The scaling with the variable N^{min} is valid much better than with x_F* . The cumulative effect corresponds to the region of limiting fragmentation for $N^{min} > 1$. Such a definition of the cumulative effect implies the hypothesis that in this range of variables the nucleus may be regarded as a noncoherent mixture of point-like constituents. The point-like similarity is necessary for the above-discussed locality of hadron-hadron interactions to be provided. The momenta of internal motion of constituents are neglected. This hypothesis is a natural generalization of the parton model[11].

We illustrate this model by a simple example. It is analogous to the impulse approximation of nuclear physics: due to relativistic time dilation the characteristic times of internal dynamics of the system turn out to be much longer than the collision time. The cross section of collision with a hadron is expressed in terms of the probability $w(x)$ of discovering inside the hadron moving with a momentum P a constituent (parton) possessing a fraction of the momentum equal to xP for $x < 1$ and in terms of the cross section for collision with a free constituent. In early related papers the process of deep inelatic scattering of electrons on protons $e + p \to e + x$ was discussed. Let q be the proton four-momentum transferred to a parton. The momentum conservation law reads:

$$x\,p + q = p' \tag{2.2}$$

where p' is the parton momentum after collision.

Squaring we have

$$q^2 + 2xq \cdot p + (xp)^2 = p'^2 \ .$$

*The fact that various authors use different variables and different coordinate systems leads sometimes to a confusion. In particular, the Berkeley physicists assert that they come to a disagreement comparing their data on the inclusive cross sections with the appropriate data of the Dubna physicists. This disagreement is explained by the fact that the Berkeley physicists use as the variable $x_F = p_z^*/(p_z^*)^{max}$, where p_z^* is the momentum in c.m.s., while the Dubna physicists use N^{min}, that is, the comparison is made in different coordinate frames. In order to make a right comparison the transition from one coordinate system to the other is needed. This wrong assertion was repeated[9] in spite of our explanations[10].

Putting $(xp)^2 = p'^2$, in accordance with interaction instantaneousness, we have $x = -\frac{q^2}{2(p\cdot q)}$ and, respectively, $w(x) = w(-\frac{q^2}{2(q\cdot p)})$. We have obtained the scale invariance of the cross section of interaction of a virtual photon with a hadron or the famous Bjorken scaling. The quantity x is the Bjorken variable . This naive model has recently been given grounds with appropriate restrictions and corrections in a theory to which we refer as quantum chromodynamics (QCD). QCD is a serious attempt to construct a consistent dynamic theory of quark matter.

In 1971 we generalized the parton model to nuclei and predicted the cumulative effect[12]. The generalization is based on the simple remark that compared to the elementary particle the nucleus can more successfully be thought of as a parton gas in the relativistic energy region since the life-times of the virtual state of the nucleus as a set of free nucleons are much longer than the life-time of the nucleon as a set of partons. It is just the ratio of the collision time to these life-times that serves as a smallness parameter which defines whether the impulse approximation (or the parton model) is applicable. At the same time, it follows from this consideration that the probability of transferring large momenta to nuclei is defined by the probability of detecting in the nucleus a parton with a momentum equal to that of a nucleon group.

Thus, the parton model contains scale invariance but does not predict the magnitude of the probability for collectivization of the parton constituents of different nucleons and for production of "large" partons, i.e. partons with momenta much larger than that per one nucleon of the nucleus.

In this connection we paid our attention to the region of the variables of relativistic nuclear collisions in which the parton model is applicable but, according to its general conception, requires that the parton with a momentum which is larger than the momentum per one nucleon of the nucleus should participate in the collision. The experimental discovery of particles in this region, that is, in the part of the region of limiting fragmentation of nuclei in which the produced particles have $N^{min} > 1$ (eq. (2.1)), provides evidence for the existence of many baryon configurations participating in the formation of one parton.

We go over to the discussion of related experimental facts, but first we consider our model in more detail. In accordance with the parton model we consider the one-particle distribution ρ in the region of limiting fragmentation of nucleus as a superposition of one particle distributions ρ_N describing the limiting fragmentation of objects of mass Nm_p inside the nucleus:

$$\rho_I^{II}(y_1 - y_I, r_1) = \sum_N P_N \cdot \rho_N(y_1 - y_I, r_1) \qquad (2.3)$$

The following properties of the cumulative effect attract our attention even if we do not recourse to further assumptions about the probability P_N of finding a constituent of mass $N m_p$ inside the nucleus and about an explicit form of ρ_N :

1. The dependence of ρ_I^{II} on the particle II properties should practically be absent owing to the general property of hadron matter, the limiting fragmentation. Resulting from the latter, the majority of the below discussed experimental facts was obtained from the particle production on nuclei by proton to the backward hemisphere mainly for the 180° angle or $r_1 = 0$ (see Section I) rather than from the particle production by relativistic heavy ions.

2. We introduce in eq.(2.3), instead of the rapidity difference $y_1 - y_I$, a quantity which describes explicitly the cumulativity of the observed process, the so-called cumulative number:

and rewrite eq. (2.3) in the form

$$\rho_I^{II}(N^{min}, r_1) = \sum_N P_N \rho_N(N^{min}, r_1) \tag{2.4}$$

According to the definition of ρ_N and the kinematic limits (1.17)

$$\rho_N(N^{min}, r_1) = 0 \qquad \text{at} \qquad N < N^{min} .$$

Hence, it is clear that N^{min} specifies in eq. (2.4) the lower limit of summation or integration, the latter - if it is supposed that N takes continuous values, as is assumed in parton models. It is doubtful that many nucleons can gather together in a small cumulation volume. Consequently, P_N must be a sharply decreasing function on N and it can be supposed

$$\rho_I^{II}(N^{min}, r_1) \approx P_{N^{min}} \rho_{N^{min}}(N^{min}, r_1) \tag{2.5}$$

Thus, according to our model, the basic quantity describing the cumulative effect $\rho_I^{II}(N^{min}, r_1)$ can be approximated by a rapidly decreasing function, for example, by an exponential

$$\rho = C \cdot \exp[-aN^{min}] \tag{2.6}$$

where a and C are the quantities practically independent of the properties of particle or nucleus II in the region of limiting fragmentation of nucleus I . Such kind universal energy dependences have been discovered. The experimental data on cumulative pion production, when the fragmenting nucleus is at rest, were often presented in the form:

$$E\frac{d\sigma}{d\vec{p}} = C \cdot \exp\left[-\frac{T_\pi}{T_\pi^\circ}\right] \tag{2.7}$$

where T_π is the kinetic energy of cumulative pions. According to eq. (2.1), between the variables T_π and N^{min} (for $\theta = 180°$) there exists a linear dependence $T_\pi = N^{min}\frac{m_p}{2} - m_\pi$ and it is not difficult to compare eq.(2.7) with the suggested dependence (2.6). Following our model, it is not appropriate to present the data on cumulative production of protons and other heavy particles in the form

$$E\frac{d\sigma}{d\vec{p}} = C \cdot \exp\left[-\frac{T}{T_0}\right] \tag{2.8}$$

or

$$E\frac{d\sigma}{d\vec{p}} = C \cdot \exp[-Bp^2] \tag{2.9}$$

which is often used in literature, since in this case

$$N^{min} = \frac{E - p_z}{m_p}\Big|_{\theta=180°} = 1 + \frac{T_p + p}{m_p} \approx 1 + \frac{p^2 + 2m_p p}{2m_p^2}$$

It should be noted that the cumulative baryon systems are rather slow, for them $|y_1 - y_I|$ reaches 2 for the cumulative numbers larger than 8 (see eq.(2.1)). Consequently, almost all the cumulative protons, deuterons and tritons studied up to the present time lie in the region of SRO and are strongly affected by the interactions in the final state.

3. The quantities ρ_N are supposed to be identical for different nuclei, because they are defined by the properties of nuclear metter at short distances. The dependence of ρ_I^{II} on the properties of nucleus I , especially on the atomic weight A_I , is contained in P_N which characterizes the probability of finding in the nucleus

a baryon configuration consisting of N nucleons at short distances. It was clear from the very beginning that it is important to study the A_I dependence of the cumulative effect to which we will pay a special attention.

The order of magnitude of the cumulative effect cross section and the A_I dependence of ρ_I^{II} were predicted on the basis of the following assumptions:

a. The constituents move in a nucleus in an uncorrelated manner, that is, P_N is described by the binomial distribution:

$$\frac{A_I!}{N!(A_I-N)!} q^N (1-q)^{A_I-N}$$

or the Poisson distribution:

$$P_N = \frac{e^{-\bar{N}}}{N!} \bar{N}^N \tag{2.10}$$

where

$$\bar{N} \approx A_I \cdot q$$

Here q is the probability for the constituent to fall into the cumulative region where the nucleons lose their individuality. This region was taken to be R ≈ 0.5 ÷ 0.7 fm, which will be seen to be well confirmed experimentally.

b. The multinucleon claster occupying the cumulation region possesses the properties of a usual hadron. Therefore in order to evaluate ρ_I^{II}, ρ_N can be taken from the data on particle production in pp collisions. The quantity q was defined: i) as a probability for a nucleon to fall into a volume $\frac{4\pi}{3}R^3 << \frac{4\pi}{3}r_0^3 A_I$, where A_I is the atomic weight.

$\frac{4\pi}{3} r_0^3$ is the volume per one nucleon

$$q = (\frac{R}{r_0 A^{1/3}}) = (\frac{R}{r_0})^3 \frac{1}{A} \tag{2.11}$$

and ii) as a probability for a nucleon to fall into a region $2\pi R^2$ ("tube")

$$q = (\frac{R}{r_0})^2 \frac{1}{A^{2/3}} \tag{2.12}$$

The first case corresponds to the volume density fluctuation. It is easily seen that we have an approximate independence of P_N of the atomic mass A of a fragmenting nucleus:

$$P_N \sim \frac{A!}{N!(A-N)!} q^N \,a\, \frac{1}{N!}(1-\frac{1}{A})(1-\frac{2}{A})... \approx \text{const}(A)$$

For the Poisson distribution

$$\bar{N} \approx qA = (\frac{R}{r_0})^3 = \text{const}(A) \tag{2.13}$$

A possible existence of the volume fluctuations of the nuclear matter density was indicated as early as in 1957[13] when explaining the knocking out of deuterons by 600 MeV protons[14]. The second case corresponds to the interaction of a target with an effective part of the nucleus centred around the path of the target throughout the nucleus. We have here a nontrivial A dependence:

$$P \approx \frac{A!}{N!(A-N)!} q^N \approx \frac{1}{N!} \frac{A!}{(A-N)!} A^{2N/3} = \frac{A^{N/3}}{N!}(1-\frac{1}{A})... \tag{2.14}$$

For the Poisson distribution we have a similar dependence, since in this case $\bar{N} = (\frac{R}{r_0})^2 A^{1/3}$ and $\bar{N}^N = (\frac{R}{r_0})^{2N} A^{N/3}$. When the cumulative number N increases by unity there arises an additional factor $A^{1/3}$. The two mentioned cases give strongly different A dependences of ρ_I^{II} : in the first one ρ_I^{II} is independent of A_I, and in the second case, according to eq. (2.5), we have

$$\rho \propto P_{N^{min}} \propto A^{N^{min}/3}, \qquad (2.15)$$

that is, ρ_I^{II} must depend on the atomic weight in an exponential manner A^m ; where m increases monotonously with cumulative number N^{min}. According to these considerations, the cumulative production cross section must behave like $A_I^{\frac{N^{min}}{3}+\frac{2}{3}}$. In the region $N^{min}=1$ the power exponent for A_I is expected to be larger than unity. The A dependence of the cumulative effect of such a kind has been found in the experiments of the Stavinsky's team[15]. A similar A dependence has been detected by the Cronin's team[16] in experiments on production of particles with large perpendicular momenta $p_\perp = r$ by 300 GeV protons. Cronin has obtained the A dependence of the cross sections for these processes of the kind $A^{\alpha(r)}$. It was found that $\alpha(r)$ is a monotonous function increasing with r which exceeds unity for $r > 2$ GeV outside the limits of errors. This analogy of the processes should be expected starting from the developed ideas about the effective interaction of a nucleon group in the region of variables where the kinematics of the problem requires collisions with a massive object.

The formulation of the problem in the case of the cumulative effect has the advantage that the studied variable ranges exclude successively one-, two- and subsequent nucleon collisions. In the Cronin's team experiments there is no strong kinematic forbiddenness.

Below we give a brief survey of the experimental data on the cumulative effect. The reviews[17] and[18] give a more complete idea about the experimental studies in the field of cumulative particle production. We mainly use the data of the Dubna team guided by Stavinsky which was first to discover the cumulative meson production in 1971. Then during eight years this team was studying the cumulative effect varying both the colliding objects and the cumulative particles. The following nuclei were used in the latter experiments: d, ^{4}He, ^{6}Li, ^{7}Li, ^{12}C, Al, Cu, ^{112}Sn, ^{124}Sn, ^{144}Sm, ^{154}Sm, ^{182}W, ^{186}W, Pb, U. As the cumulative particles one used: π, p, d, T, ^{4}He, ^{3}He. Figs. 3,4,5 and 6 present data on the cumulative meson production on nuclei in reactions $p + A_I \to \pi(180°)$ and $d + A_I \to \pi(180°)$ for different proton and deuteron momenta. The $E\frac{d\sigma}{d\vec{p}}$ values for different nuclei normalized to the atomic weight of a fragmenting nucleus are given as a function of the pion kinetic energy. The cumulative number $N^{min}=1$ corresponds to T_π = 270 MeV and $T^{min}=2$ corresponds to T_π = 629 MeV. Figs. 3-6 well illustrate the universal character of the dependence of the invariant cross sections on N^{min} (or T_π) and on A_I.

In order to prove experimentally the existence of the cumulative effect it was very important to show that really we were dealing with the limiting fragmentation of nuclei. To this end, the data presented in figs. 3-6 were treated by means of eq. (2.7) with the aim to extract the parameter T_π°. In fig.7 this parameter is presented as a function of the momentum per nucleon of colliding nuclei p_{II}°. Starting with the momenta higher than $p_{II}^\circ \approx 4$ GeV/c, T_π° is seen to be no longer dependent on p_{II}°. This fact is in good agreement with the SRO model and the estimate (1.14) defining the region where the limiting fragmentation of nuclei begins. The limiting value of T_π° is found to be equal to 60 MeV. The paper by Fujioka et al[19] submitted to the Tokyo conference contains the results of measurement of T_π° for p_I° equal to 12.6 - 28.5 GeV/c. In this energy range T_π° was found to be constant and equal to 60-65 MeV.

Similar results were obtained for the case when the cumulative particles were taken to be protons and other baryon systems. Figs. 8,9 and 10 give the experimental data

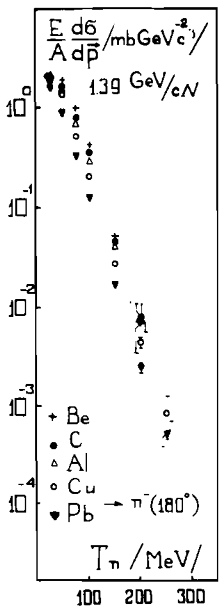

Fig. 3

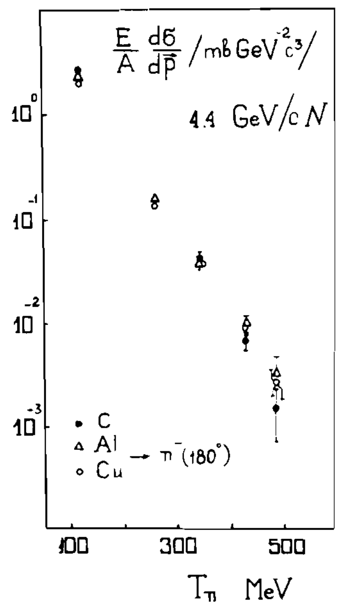

Fig. 4

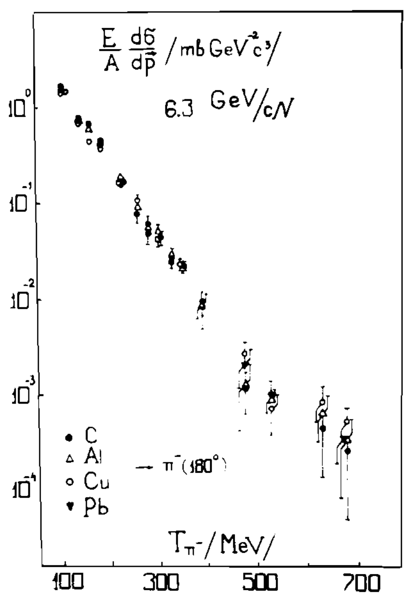

Fig. 5

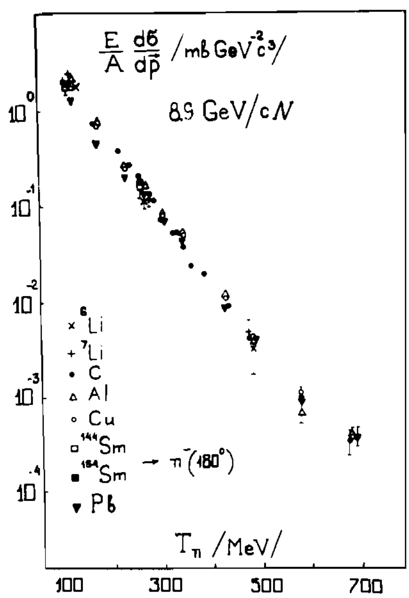

Fig. 6

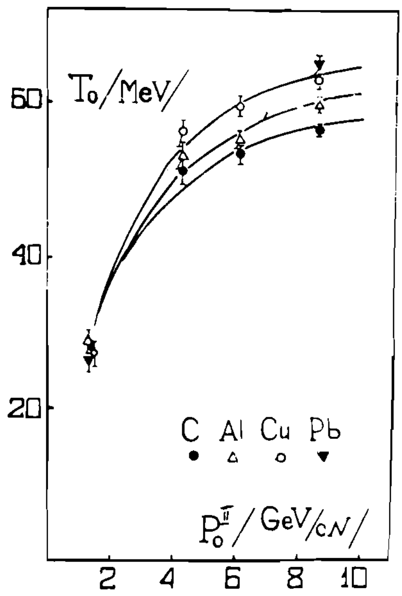

Fig. 7

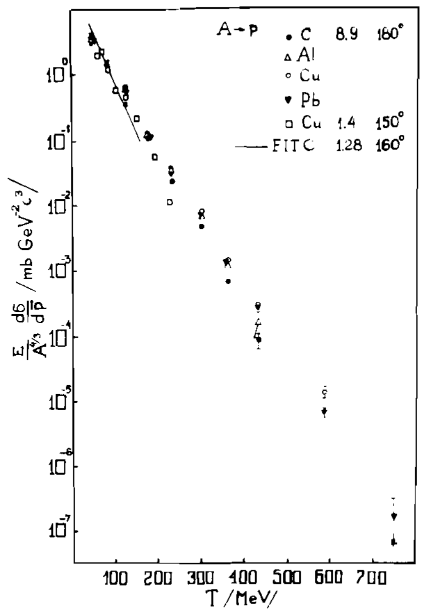

Fig. 8

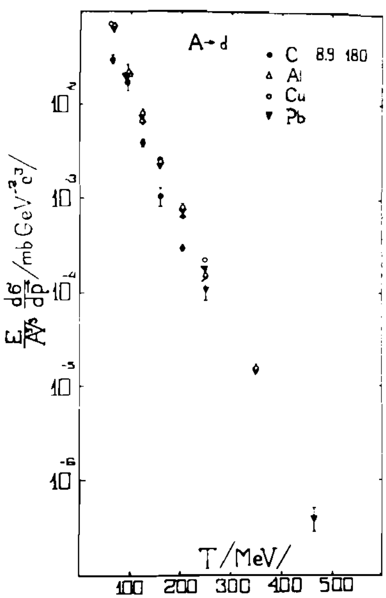

Fig. 9

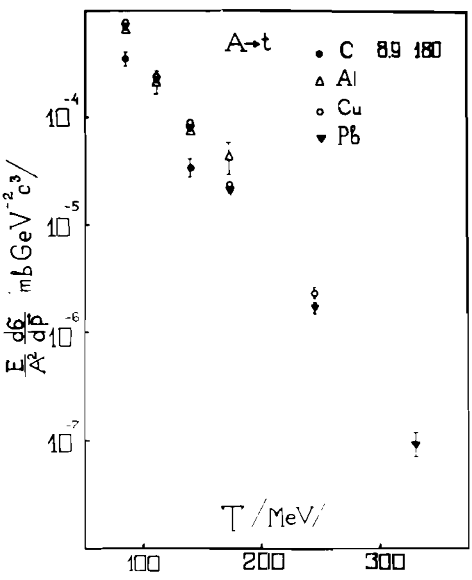

Fig. 10

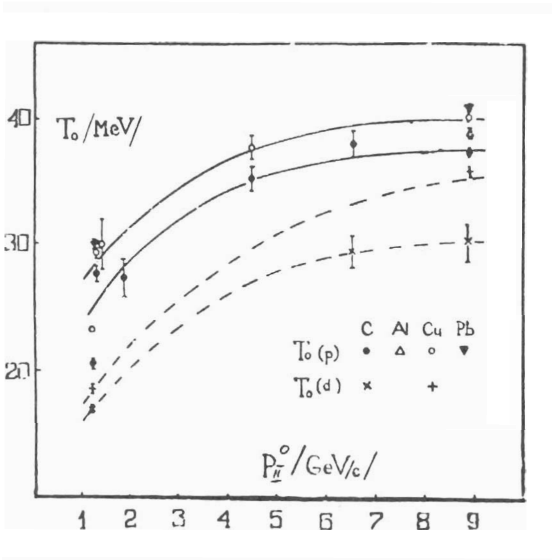

Fig. 11

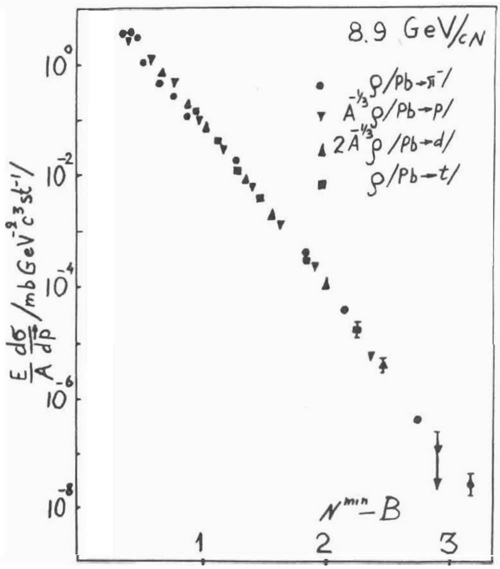

Fig. 12

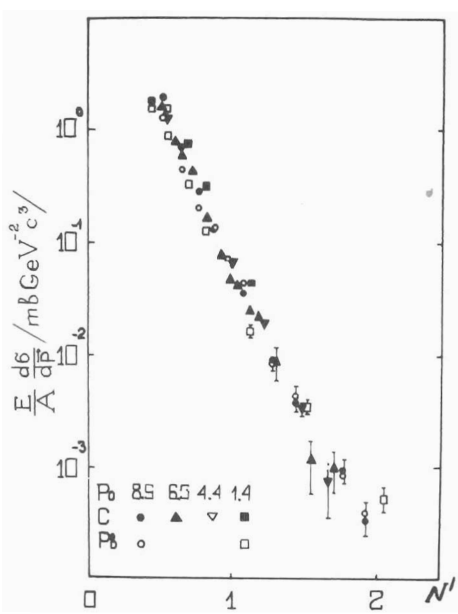

Fig. 13

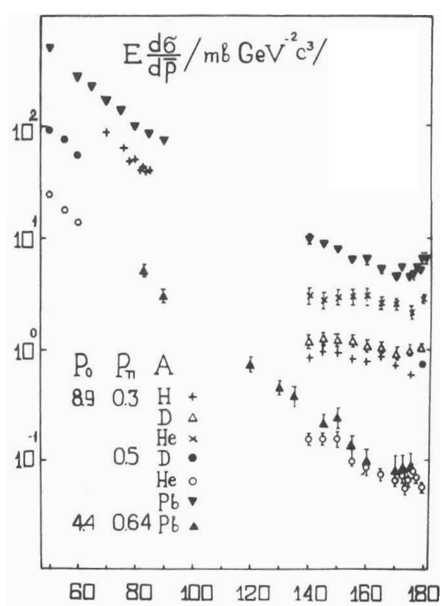

Fig. 14

of the Stavinsky's team on the kinetic energy dependence of cumulative baryon systems: protons, deuterons and tritons, respectively, bombarded by the nuclei of carbon, aluminium, copper and lead with a momentum of 8.9 GeV/c. The fragments are detected at an angle of 180° to the primary protons. The inclusive cross sections are normalized to $A^{4/3}$ for protons, $A^{5/3}$ for deuterons and A^2 for the tritium nuclei. As in the case of cumulative pions, the spectra are of a universal character. For protons the universal exponential describes changes in the cross section by eight orders of magnitude.

Fig.11 gives the T_0 values for cumulative protons obtained by the treatment of the experimental data by eq. (2.7) for various fragmenting nuclei (C, Al, Cu, Pb) depending on the momentum of primary protons: 1.22 GeV/c (180°)[20], 1.28 GeV/c (140°)[21], 1.39 GeV/c (150°)[22], 1.86, 4.50, 6.57 GeV/c (137°)[23,24], 8.9 GeV/c (180°)[25]. As is seen from fig.11, the parameter T_0 increases with increasing energy of colliding particles. In just the same way as in the case of cumulative pions, the limiting fragmentation begins at $p^o_{II} = 4$ GeV/c, where T_0 reaches 40 MeV. The Japanese physicists, Fujioka et al.[19], measured the parameter T_0 in the energy interval of primary protons from 4 to 205 GeV/c which was found to be equal to 40-45 MeV and to be in excellent agreement with the bulk of data at relatively low energies and with our conclusion that the limiting fragmentation begins at $p^o_{II} =$ 4 GeV/c.

The parameter T^o is often called "temperature". From the point of view of our model the cumulative effect is mainly the result of one hard collision and the notion of temperature is not applicable in this case. If the experimental data are described by eq. (2.6) and the parameter a is taken to be equal to $1/\bar{N}$, where $\bar{N}$ is given by $\bar{N} = (R/r_0)^3 \approx 0.16$, then there arises a surprising possibility of describing all the cumulative spectra by this parameter alone and thereby explaining the quantity T_0. R is close in its magnitude to the confinement radius. In order to illustrate to what extent such a description is successful we give in fig.12 new data

parametrized by $E\frac{d\sigma}{d\vec{p}} = C \cdot \exp[-\frac{N}{\bar{N}}]$ for $\bar{N} = 0.16$ obtained by the Stavinsky's team.

The discussed parametrization describes the change of different cross sections by nine orders of magnitude up to high cumulativity orders. It is difficult to conceive such a coincidence in the light of the above remark about the essential role of the final states interaction effects for baryon systems. In ref.[18] an attempt has been made to improve this parametrization so that it describes the region of low energies too. It is supposed in this approach that at low energies the cumulative region increases by the wave length of colliding particles

$$\bar{N} = \left(\frac{R+\lambda}{r_0}\right)^3 \qquad \text{where} \qquad \lambda = \frac{1}{p^o_{II}}$$

This is equivalent to the replacement of the variable N^{min} by

$$N' = N^{min}\left(\frac{R}{R+\lambda}\right)^3 . \tag{2.16}$$

Fig.13 gives the experimental data on the cross section of the reaction $p+A \to \pi\,(180^o)$ constructed as a function of the variable N' for different p^o_{II} values. Eq. (2.6) describes also the angular distributions of the cumulative particles to the discussion of which we are just proceeding.

Fig.14 shows the experimental data on the production cross section $E\frac{d\sigma}{d\vec{p}}$ for cumulative π^o mesons with momenta 300, 500 and 640 MeV/c as a function of the angle θ_π . θ_π is the angle between the momenta of the primary proton and a pion produced in the reaction. It became possible to measure the angular distributions of cumulative particles after a rotating magnetic spectrometer DISK operating on the Dubna Synchrophasotron beams had been created by the Stavinsky's team.

It is seen from fig.14 that the θ_π dependences of the pion production cross section for the helium nucleus (the pion momentum is 500 MeV/c ϕ) and for the lead nucleus (the pion momentum is 640 MeV/c ▲) are practically the same. A remarkable similarity of the angular distributions of the lightest and heavy nucleus fragmentation excludes the cumulative effect models based on the hypothesis about successive particle interactions inside the nucleus. We consider to what extent the simple model of the type (2.6) taking into account the correction (2.16) is valid for the description of the angular distributions of cumulative particles in a wide energy range of nuclear collisions. In fig.15 taken from ref.[18] we give the same experimental data as in fig.14 but presented as a function of the cumulative number N' . The same figure presents the experimental data from ref.[22] for fragmentation of the lead nucleus with production of negative pions emitted at an angle of 90°- 150°. The straight line corresponds to the experimental data on the energy dependence of the cross section for an angle of $\theta_\pi = 180^\circ$ taken from fig.13. Taking into account the fact that we attempt to describe the bulk of experimental data on the energy and angular dependence by means of the simple formula (2.6) which contains no fit parameters (the parameter a is determined from $\bar{N}$ according to the model) the agreement must be admitted to be satisfactoty. In paper[26] the data on the cumulative production of Λ hyperons in the reactions $\pi + C \to \Lambda$ and $n + C \to \Lambda$ which contain also measurements of the angular distributions are treated with the aid of eq. (2.6). The authours come to the conclusion that this formula well describes both the angular and momentum distributions of cumulative Λ hyperons. The study of the cumulative production of Λ particles is especially interesting since it enables us to investigate the cumulative particle polarization which is very sensitive to theoretical models. At present there are data[26,27] on the polarization of cumulative Λ particles in the collision of pions and neutrons with nuclei which will briefly be discussed in the section devoted to the theory of the cumulative effect.

The cumulative production of baryon systems was studied by many authors. Leksin and his co-workers[28] were the first to pay attention to the universal character of the spectra of baryon systems emitted to the backward hemisphere. The quantity B in the formula $\rho = C \cdot \exp[-Bp^2]$ which describes the proton production on nuclei at large angles was found to be independent of the energy and properties of bombarding particles in accordance with the limiting fragmentation properties. The empirical rule-hypothesis according to which the inclusive cross section normalized to the total cross section is a universal function which is independent of the type of the interaction and the collision energy was called by Leksin nuclear scaling. A large number of papers of the Institute of Theoretical and Experimental Physics[29] is devoted to the study of the nuclear scaling hypothesis.

An interesting confirmation of the universal dependence of the type (2.6) was obtained in Batavia where the cross section for proton production in the backward hemisphere in the antineutrino-neon interactions was studied (see the review by Nezrick[30]) and on the Erevan accelerator where photons were used as particles II. Systematic experimental studies of the cumulative proton spectra were performed on the Erevan electron accelerator by the Egijan's team[31]. Fig.16 gives the angular dependence of the parameter $B = \frac{1}{2 m_p T_0}$, where m_p is the proton mass obtained in reactions of limiting fragmentation of nuclei ^{12}C (◆), ^{63}Cu (▲) and ^{208}Pb (◇) indiced by gamma quanta with an energy $(E_\gamma)_{max} = 4.5$ GeV from ref.[31]. The data marked by (▲) are taken from ref.[32] and are relative to an energy $(E_\gamma)_{max} = 1.2$ GeV. It is worth noting that the T_0 value obtained in this case for angles close to 180° is in satisfactory agreement with the discussed above ($T_0 \approx 40$-50 MeV).

According to SRO the limiting fragmentation of nuclei under the action of pions, gamma quanta and neutrinos begins at rather lower energies than that under the action of baryon systems. For proton-nucleus collisions the parameter T° assumes the asymptotic value at a proton momentum of 4 GeV/c (figs.7 and 11) whereas for pion-nucleus collisions at a pion momentum as small as 1.5 GeV/c (fig.17). Fig.17 is a result of conversion[18] of the data on the B parameter obtained by the magnetic spectrometer ISTRA-2 of the Institute of Theoretical and Experimental Physics.

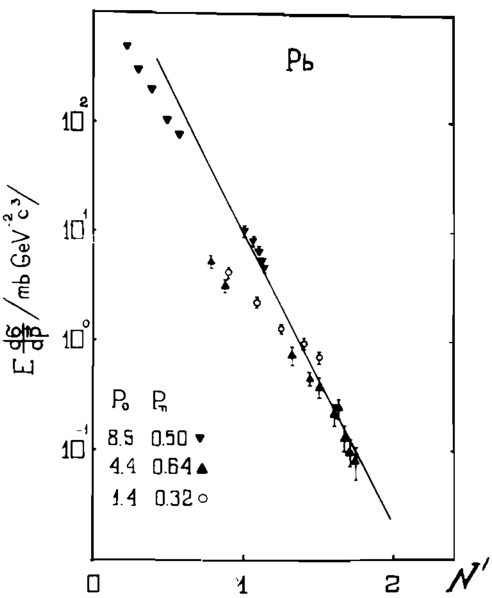

Fig. 15

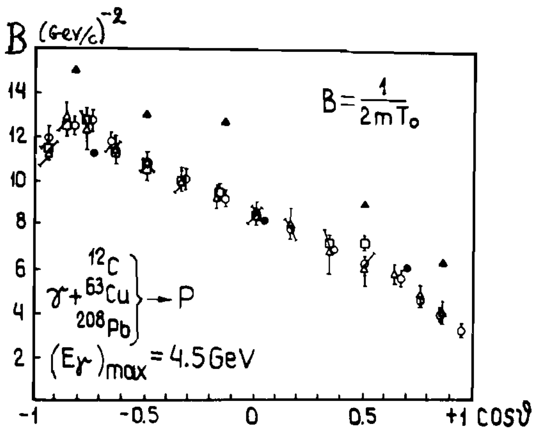

Fig. 16

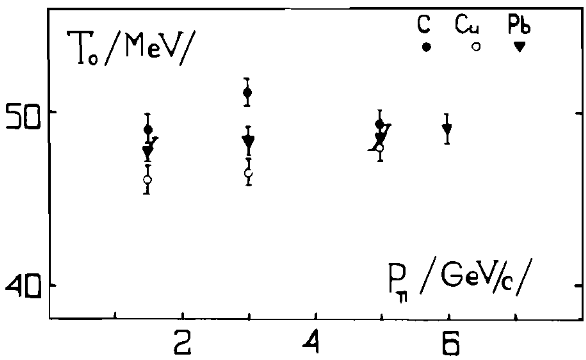

Fig. 17

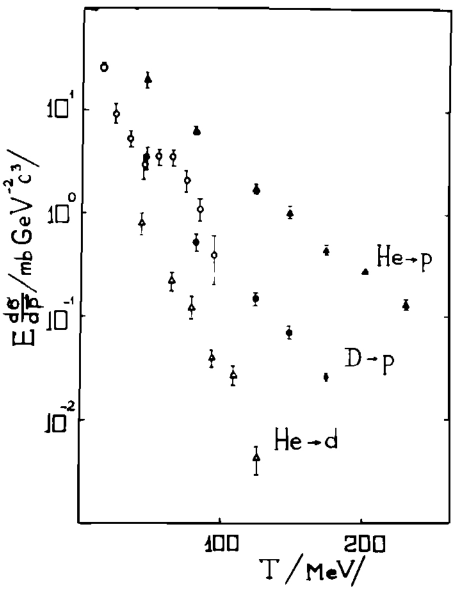

Fig. 18

The beginning of the limiting fragmentation of nuclei at low energies under the action of light particles confirms the local character of interactions of relativistic nuclei in the abovementioned sense. It is necessary to stress the particular importance of the SRO correlations for the case when the cumulative particles are baryon systems. The effect is in this case complicated by the interactions in the final state because the difference of the rapidities of the cumulative particle and fragmenting nucleus is small enough. This is also confirmed by experimental evidence.

Firstly, a sharp increase of the A dependence of the yields of cumulative baryon systems observed by the Stavinsky' team (fig.8,9, and 10) may be thought of as a consequence of the interaction in the final state. In fact, according to the experimental data presented in figs. 8,9 and 10 the A dependence increase behaves in the following manner: the factor $A^{1/3}$ is added to each unit of the baryon number B of a cumulative particle:

$$\begin{aligned} \sigma(A \to \pi) &\propto A \\ \sigma(A \to p) &\propto A^{4/3} \\ \sigma(A \to d) &\propto A^{5/3} \\ \sigma(A \to t) &\propto A^{2} \end{aligned} \qquad (2.17)$$

If we apply our model of the cumulative effect (calculation of P_N in eq.(2.15)) not only to the initial stage of the process, but also to the description of the interaction of a cumulative parton in the final state, then eq.(2.15) gives just the mentioned increase of the A dependence by a factor $A^{B/3}$. In this case the increase is explained by the thickness of the nuclear metter through which the cumulative parton passes. If our explanation reflects the real happening then with increasing cumulative particle energy (high cumulative orders) A dependences must weaken according to the SRO model.

Secondly, a peculiarity is observed for $T \approx 60$ MeV (fig.18) in the kinetic energy distribution of cumulative baryons (protons,Λparticles) emitted in the backward hemisphere. This peculiarity can simply be explained as the effect of final state interaction of two baryon systems. It was well studied in the reaction $d+p \to p(pn)$ on the basis of the data obtained by the Glagolev's team on a hydrogen bubble chamber irradiated by a relativistic deuteron beam of the Dubna Synchrophasotron.

In connection with the foregoing I conceive the cumulative production of baryon systems as a phenomenon strongly complicated by the final state interaction effect which is neglected by the theories based on the idea about hard collisions (see below).

III. Development of the Models of Relativistic Nuclear Physics

In the region of multibaryon phenomena specified by the condition (1.1) $\frac{\vec{p}^2}{m^2} \gg 1$ of importance is the production of new particles. The basic concept of nuclear physics, the wave function of a finite number of baryons involved in the problem, is found to be invalid.

The relativistic description of multiparticle states encounters the following difficulties: i) We have to deal with a variable number of particles and, consequently, with an infinite number of degrees of freedom; ii) A many-time formalism is needed; iii) It is impossible to separate the contributions of particles and antiparticles in a relativistic invariant manner, to separate the internal motion from the motion of the composite system as whole.

In addition, it is necessary to take into consideration advances in hadron physics. Namely, one cannot neglect the finite sizes of nucleons, their internal quark structure, it is necessary to take into account the finiteness of the characteristic time of formation of hadrons out of partons. According to the existing approaches which have been confirmed experimentally and have been given theoretical grounds the hadron interaction in the range of short wave lengths (large momentum transfers) has a local character. It proceeds in a region the dimensions of which are far smaller than the nucleon sizes. The emergence of hadrons "from a point", their growth up to "adult" sizes require a certain time during which a "rew-born" hadron can cover a long distance without undergoing secondary interactions. In connection with the foregoing, the attempts to expalin the cumulative effect by the Fermi motion or rescattering of particles inside the nucleus seem to me to have no grounds. For the last eight years from the moment of discovery of the cumulative meson production the number of papers in which these attempts were undertaken has strongly increased. I do not intend to discuss them here. The possibility of explaining the particle production in the cumulative region on the basis of the Fermi motion was especially persistently upheld by the Berkeley physicists[8,9]. The asssumption about "long tails" of the Fermi distributions is in contradiction with what we presently know about the nucleons. Furthermore, the abovementioned strong A dependences and other particular features of the cumulative effect cannot, on principle, be explained by the Fermi-motion.

The description of states in the Fock space is the most adequate one to the relativistic nuclear physics, since it can be used to define states with a variable number of particles and, at the same time, allows an interpretation similar to that of the wave functions in nonrelativistic theory.

The Fock column defined on the hyperplane $t=0$ ("equal time") in the coordinate space is

$$\Phi = \begin{Bmatrix} \Psi_1(\vec{x}_1) \\ \Psi_2(\vec{x}_1,\vec{x}_2) \\ \cdots \\ \Psi_n(\vec{x}_1,\dots,\vec{x}_n) \\ \cdots \end{Bmatrix} \qquad (3.1)$$

The squared functions $\Psi_n(\vec{x}_1,\dots,\vec{x}_n)$ have the meaning of the probability density for n particles to be found in a system. It is not difficult to show (e.g.,ref.[33]) that in the nonrelativistic case when the Hamiltonian of the system commutes with the particle number operator the Fock space desintegrates into subspaces. Each subspace has then the Schroedinger equation for an approapriate number of particles. In the nonrelativistic limit the position of the particles has no time to change essentially during the time difference between the events,therefore,the wave function is expressed through itself. While in the relativistic case when the particle production and annihilation can occur the Hamiltonian and the momentum operators do not commute with the particle number operator, neither do the Lorentz transformation operators. All these operators are expressed in terms of a lagrangian which contains an interaction changing the number of particles. This means that in the Lorentz transformation the lines of the Fock column get mixed up and, in different coordinate frames, the composition of a moving object, e.g. of a nucleus, will be different.

The number of the particles in a system depends on the momentum with which it moves. In this connection of particular importance is the concept of the coordinate frame moving with infinitely large momentum[34], IMF. For a certain class of theories, a composite object in this frame becomes a set of almost noninteracting constituents. This idea underlies the parton models[11] which are successfully applied to the collisions of "elementary" hadrons[35]. It is obvious that, according to the principle of relativity, the physics of the problem must be independent of the choice of the coordinate frame and the results must have a relativistic invariant form. The choice of the coordinate frame enables us only to make the problem simpler,to distinguish explicitly physical assumptions and restrictions. It is no coincidence that in nonrelativistic nuclear physics we use the rest frame rather than the coordinate frame in which the nucleus moves with a velocity close to that of light. The application of the IMF is found to be useful when the properties of a system make it possible to employ methods analogous to the impulse approximation, namely, when the time of collision of an external object with the system is small compared to the times characterizing the internal motion of the system.

One of the essential troubles of relativistic description consists in that the internal motion of a composite system is not separable from the c.m. motion. The problem becomes, however, simpler if theories are formulated on the light cone (see below).

The time development of a system is defined by the total energy which for a system of free particles is defined as

$$E = \sum_{i=1}^{n} \sqrt{\vec{p}_i^{\,2} + m_i^2} \tag{3.2}$$

Let the motion along the axis z satisfy the basic criterion (1.1), then

$$\begin{aligned} E &= \sum_{i=1}^{n} \sqrt{p_{iz}^2 + r_i^2 + m_i^2} \approx \sum_{i=1}^{n} \left(p_{iz} + \frac{1}{2}\,\frac{r_i^2 + m_i^2}{p_{iz}}\right) = \\ &= P_z + \sum_{i=1}^{n} \frac{r_i^2 + m_i^2}{2p_{iz}}, \end{aligned} \tag{3.3}$$

where P is the total momentum of the system. It is seen that in a coordinate frame where $P_z \to \infty$ it is possible to divide the motion of the system into the motion as a whole and the internal motion. The Schroedinger equation is then written as:

$$i\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)|\Psi> = (E - P_z)|\Psi> \tag{3.4}$$

The effective Hamiltonian which corresponds to the transverse and relative motion, according to eq. (3.3), is written as

$$E - P_z = \sum_{i=1}^{n} \frac{r_i^2 + m_i^2}{2p_{iz}}\,.$$

This Hamiltonian decreases as $\sim \frac{1}{p_{iz}}$ at $p_{iz} \to \infty$. This is just a reflection of time dilation.

It should be stressed that our approach is based on the following hypothesis: there exists such p_{iz} that all the internal and transverse momenta are much smaller than this quantity. In particular, in the IMF there are no particles with negative p_{iz} components.

It is convenient to work in the IMF with the light cone coordinates (τ , x , y , ζ) which are linked with the ordinary coordinates in the following manner:

$$\tau = \frac{1}{\sqrt{2}}(t' + z'); \quad x = x'; \quad y = y'; \quad \zeta = \frac{1}{\sqrt{2}}(t' - z').$$

The energy-momentum variables conjugate to the latter are obviously found from

$$p_\mu x^\mu = H\tau + \eta\zeta + p_x x + p_y y \tag{3.5}$$

from where

$$H = \frac{1}{\sqrt{2}}(E - p_z); \quad p_x = p'_x; \quad p_y = p'_y; \quad \eta = \frac{1}{\sqrt{2}}(E + p_z)$$

It is convenient to express the transition from the ordinary coordinate frame to the IMF in terms of the hyperbolic angle ω between the time axes of these systems

$$\begin{aligned} p_x &= p'_x; \quad p_y = p'_y \\ p_z &= p'_z \operatorname{ch}\omega + p'_t \operatorname{sh}\omega \\ p_t &= p'_z \operatorname{sh}\omega + p'_t \operatorname{ch}\omega \end{aligned}$$

The case we are considering corresponds to $\omega \to \infty$

or

$$\begin{cases} \mathrm{ch}\,\omega \to \frac{1}{2} e^{\omega} \\ \mathrm{sh}\,\omega \to \frac{1}{2} e^{\omega} \end{cases}$$

and

$$x = x'; \; y = y'$$

$$z = \frac{1}{2}(z' + t')\, e^{\omega}$$

$$t = \frac{1}{2}(z' + t')\, e^{\omega} \; .$$

It is seen that the space hypersurface for the IMF corresponding to $t = 0$ is $z'+t'=0$ in the ordinary coordinate frame. This surface is tangent to the light cone.

Boosts along the z-axis in variables η and r are a pure rescaling of the η's

$$\eta \to e^{\omega}\, \eta \; , \quad r \to r \tag{3.6}$$

Since the theory must be invariant under such boosts, the physically meaningful quantities are ratios of the η's.

We consider infinitesimal rotations around the axes x and y

$$\begin{aligned} p_y &\to p_y \cos\epsilon + p_z \sin\epsilon \\ p_z &\to -p_y \sin\epsilon + p_z \cos\epsilon \; . \end{aligned} \tag{3.7}$$

In terms of the variables r and η we have

$$\begin{aligned} r_y &\to r_y \cos\epsilon + \frac{1}{\sqrt{2}}\, \eta\, e^{\omega} \sin\epsilon \\ \eta &\to -\sqrt{2}\, r_y e^{-\omega} \sin\epsilon + \eta \cos\epsilon \; . \end{aligned} \tag{3.8}$$

The transformation in the transverse plane must be finite, from where it follows that $\epsilon \lesssim e^{-\omega}$ and the quantity $\sin\epsilon\, e^{-\omega}$ is of the second order of smallness. In other words,

$$\begin{aligned} r_y &\to r_y + V_y \eta \\ \eta &\to \eta \end{aligned} \tag{3.9}$$

where

$$V_y = \frac{1}{\sqrt{2}} e^{\omega} \cdot \epsilon \, .$$

Eq. (3.9) is analogous to the Galilean transformation, if η is an analog of the mass, and V that of the relative motion velocity. This analogy becomes still more complete if we recall the expression for the energy in the IMF

$$H = \sum_{i=1}^{n} \frac{r_i^2 + m_i^2}{2\eta_i} = i \frac{\partial}{\partial \tau} \tag{3.10}$$

The introduced notation and concepts make it possible to introduce the wave function of the multiparticle state in the IMF:

$$\Psi_{\eta p_\perp}(\eta_1, r_1; \ldots; \eta_n, r_n)$$

The invariance under the (3.6) and (3.9) group requires that all dependences on r and η occur through the variables $R_i = r_i - \frac{\eta_i}{\eta} p_\perp$ and the variables $\beta = \frac{\eta_i}{\eta}$. The wave function assumes then the form

$$\Psi_n = \Psi_n(\beta_1, \ldots, \beta_n, \ldots, \vec{R}_i \cdot \vec{R}_j, \ldots) \tag{3.11}$$

$\beta_i = \frac{p_{iz}+p_{it}}{P_z + P_z}$ is a fraction of the momentum which is carried by a subsystem (parton, nucleon, quark).

The normalization of the one-particle state vectors is usually taken as

$$\langle \eta, r \mid \eta' r' \rangle = \eta\delta(\eta-\eta')\delta^2(r-r') = \delta(\frac{\eta}{\eta'} - 1)\delta^2(r-r').$$

The integration is performed over the invariant measure $\frac{d\eta}{\eta} d^2 r$. In the introduced notation the normalization of the wave functions has the form

$$\sum_n \int \frac{d\beta_1 \dots d\beta_n}{\beta_1 \dots \beta_n} d^2 r_1 \dots d^2 r_n \Psi_n^*(\beta_1 \dots \beta_n, R_1 \dots R_n)\Psi_n(\beta_1 \dots \beta_n, R_1 \dots R_n) \times \\ \times \delta^2(\Sigma R_i)\,\delta(1 - \Sigma\beta_i) = 1. \quad (3.12)$$

The above-formulated hypothesis about the finiteness of $p_{i\perp} = r$ and, in general, of the momenta of internal motion has led us to the fact that the wave function depends on the ratio of the momenta β_i alone. Thus, this implies scale invariance.

The parton model, which we often refer to, is an approach to field theories in the IMF on the basis of the state vectors desctibed as an infinite Fock column the lines of which are specified by the functions (3.11). The facts that the vacuum fluctuations are nonessential and the processes of interactions with partons proceed on the mass shell make it possible to overcome the above-mentioned troubles of relativistic consideration and calculate the hadron processes, including the processes of relativistic nuclear physics.

The main object to which the parton model was first applied is the deep inelastic scattering of leptons on hadrons. It can be said that experiments on deep inelastic scattering of leptons $(e, \mu, \nu, \tilde{\nu})$ on partons have played the decisive role in the development of physics. The idea of these experiments consists in that by measuring the dependence of the cross sections for processes $\ell + p \to \ell + X$, where ℓ is a lepton, on the squared four-momentum transferred to the lepton q^2 we study the proton structure at distances $\sim \frac{1}{\sqrt{|q^2|}}$. If now we assume that the time τ which controls the collision process is also of the order of $\frac{1}{\sqrt{|q^2|}}$ then at $q^2 \to \infty$ the characteristic times of the internal motion $\tau_c \gg \tau$, and we are dealing with the situation described by the impulse approximation.

As far as in experiment the summation is performed over all the hadronic states X, then the description of the hadron A by means of the Fock column reduced to a description by means of $f_{A/a}(x)$, the number of partons of sort a in the hadron A with a momentum xP, where P is the total momentum of the hadron A. The quantities $f_{A/a}(x)$ are introduced phenomenologically, since we are unable to calculate them. They are normalized as

$$\sum_a \int_0^1 f_{A/a}(x)\, x\, dx = 1. \quad (3.13)$$

The cross section for the inelastic scattering of leptons $\ell + A \to \ell + X$ is of the form

$$\frac{d^2\sigma}{dq^2 d\nu} = \sum_a \int_0^1 f_{A/a}(x) \frac{d^2\sigma_a}{dq^2 d\nu} dx \quad (3.14)$$

Here $\nu = (q \cdot P) = m(E - E')$, and $\frac{d^2\sigma_a}{dq^2 d\nu}$ is the cross section for inelastic scattering of a lepton ℓ on a parton a: $\ell + a \to \ell' + a'$. For elastic scattering the conservation laws are

$$q + xP = P' \qquad \text{or} \qquad q^2 = -2x(q \cdot P)$$

and

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{d\sigma}{dq^2}\delta(\nu - \frac{q^2}{2x}) \tag{3.15}$$

$f_{A/a}(x)$ are the objects of measurements.

For a review of the study of processes proceeding with large momentum transfers the reader is referred to the lectures of Efremov at the CERN-JINR School of Physics[36].

We add some words about the validity of the orthodox parton model. We have used in the conservation law the condition that the parton momenta $k = xP$ and $P' = k + q$ are close to the mass shell. To be precise, the condition of applicability of the parton model reads in this case:

$$k^2;\ (k+q)^2 \ll q^2 \tag{3.16}$$

where k^2 and $(k+q)^2$ are the virtualities of the parton before and after the virtual photon absorption.

It is common practice to discuss the validity of the parton model in terms of the "effective coupling constant" in field theory. In quantum field theory the "interaction constant" is not, strictly speaking, a constant, it depends on the square of the momentum transfer. In particular, in electrodynamics the interaction constant α is $\frac{e^2}{4\pi} = 1/137$ only for restricted q^2. Taking into account quantum effects we are led to the fact that α is a function of q^2 which increases with q^2. The point-like nature of partons at large q^2 means that they are with a relatively small probability surrounded with a "cloud" made of virtual particles with which they can interact (gluons). The small probability of the "cloud" implies that the effective interaction constant α_s is small and, contrary to α, decreases with increasing q^2. Such a behaviour cannot be got for realistic theories with dimensionless couling constant. If we impose, in just the same way as we have done above, restrictions on the transverse momenta r, then in local and renormalizable theories it is possible to obtain a decreasing effective coupling constant, but in this case locality and Lorentz-covariance are violated. In any field theory an integration over all the virtualities is implied and the restriction (3.16) looks like an artificial one. There exists, however, a very interesting possibility[36] that the naive parton model is realized in a restricted domain:

$$\alpha_s(q^2)\log q^2/\Lambda^2 \ll 1\,. \tag{3.17}$$

The logarithmic decrease of the effective coupling constant of a quark with gluon field $\alpha_s(q^2)$ with increasing q^2 is called asymptotic freedom. This property has been discovered in QCD and is a strong argument in favour of the latter. Although the parton virtuality is in this case rather large, nevertheless, the condition (3.16) is found to be fulfilled. However, from the condition (3.17) it follows that: i) for rather large q^2 scale invariance is violated, ii) the transverse parton momenta increase with increasing q^2. Both the properties have been observed experimentally.

We have touched upon certain results of quantum field theory with the aim to show, on the one hand, that the parton model can serve as a reasonable first approximation for analysing the phenomena of large momentum transfers to multibaryon systems and, on the other hand, that the study of these phenomena bears a direct relation to the main problems of the quark theory of matter.

The above-stated apparatus is also applied for relativistic generalizations of the ordinary nuclear physics. We consider a simple example. Using the properties of the wave functions (3.11) it is not difficult to show (see ref.[35]) that a matrix element of the bilinear scalar density between two states of a composite hadron which is made of two constituents is proportional to the integral

$$\int \Psi^*(\beta_1, R_1^2)\Psi[\beta_1, (R_1+\beta_1 Q)^2] \frac{d\beta_1}{\beta_1(1-\beta_1)^2} d^2R_1, \tag{3.18}$$

where $Q = r - r'$ is the transverse momentum of an external effect. This formula has a simple nonrelativistic analog. In the initial state we have a hadron in a frame with zero transverse momentum, $\Psi(\beta_1, R_1^2)$ is the amplitude for a two-parton state with parton 1, having momentum (β_1, R_1), and parton 2 having momentum $[(1-\beta_1), -R_1]$. Then a transverse momentum Q is deposited on parton 2 to bring its momentum to $[(1-\beta_1), -R_1+Q]$. The hadron has a center of mass velocity in the transverse plane given by Q. Thus, to project the state onto the final hadronic state we must transform the wave function of the final hadron by a transformation of (3.9) type to a frame in which it moves with velocity Q. This takes each transverse momentum and translates it by amount βQ so that the argument of the final wave function is $(R_1, +\beta_1 Q)^2$.

This apparatus makes it possible to introduce the notion of wave functions describing the structure of "elementary" hadrons. These wave functions enable us to use the ordinary notions of quantum mechanics, in particular, the probability distributions. The apparatus does not provide us with a receipt for constructing the wave functions, however, after the parton momentum distributions of the type $f_{A/a}(x)$ in eq. (3.14) have been measured in one experiment, it is possible to use it for interpreting other experiments. The mentioned transition to the nonrelativistic impulse approximation on the basis of eq. (3.18) enables us to use evidence about the nonrelativistic wave functions in considering the interactions of high-energy particles with nuclei. The construction of the wave functions comes in this case to a relativistic generalization of the known wave functions of the nonrelativistic nuclear physics. For example, in ref.[37] use is made of the following relativistic generalization of the Hulthen wave function for the analysis of the reaction $d + p \to p + p + n$:

$$\phi_d = N\left[\frac{m_p^2 + r^2}{\beta(1-\beta)} - C_1\right]^{-1}\left[\frac{m_p^2 + r^2}{\beta(1-\beta)} - C_2\right]^{-1} \tag{3.19}$$

β is the same as in eq. (3.11), C_1 and C_2 are constants.

This has made it possible to describe the characteristics of interactions of relativistic deuterons with protons. As is seen from this example, contrary to pure hadron physics, in nuclear physics we know, at least, something about the wave function of a system, namely, the asymptotics of long distances and can attempt to extrapolate these functions to the relativistic domain for checking the basic ideas of hadron physics.

A consistent development of the parton model for the explanation of the cumulative effect laws has been suggested by Efremov[38,39]. The model claims to the explanation of all the main features of the cumulative effect presented in Section II, connects the cross section of this process with the cross section of production of particles with large transverse momenta r on nuclei and propose equations for describing the cumulative particle polarization.

We write down the general formula for the spin density matrix of the cumulative particle of this model[39] which is an application of the so-called theory of hard collisions and describes the process of collision of hadrons $A + B \to C + X$

$$\rho_{\mu\mu'} E\frac{d\sigma}{d\vec{p}_c} = \int dy \cdot d\Delta \cdot Q_{A/a}(\alpha) \cdot Q_{B/b}(\beta) \cdot \frac{1}{\pi}\rho_{\lambda\lambda'}\frac{d\sigma}{dt'}(s',t')D_{c/C}^{\lambda\mu\lambda'\mu'}(\gamma) \tag{3.20}$$

where $Q(x) = x f(x)$; $f_{A/a}(x)$ and $f_{B/b}(x)$ is the number of partons of sorts a and b in hadrons A and B, respectively, the quantity $\rho_{\lambda\lambda'}\frac{d\sigma}{dt'}(s',t')$ describes the process of scattering of point-like partons: $a + b \to c + d$ (comp. (3.14)); $D_{c/C}^{\lambda\mu;\lambda'\mu'}$ is the matrix of fragmentation of parton c to hadron C with a momentum fraction γ:

$$\alpha=-\frac{x_1}{\gamma}(1+e^{\Delta});\quad \beta=-\frac{x_2}{\gamma}(1+e^{-\Delta});\quad x_1=\frac{-u}{s};\quad x_2=-\frac{t}{s}$$

$$s'=\alpha\beta s;\quad t'=\frac{\alpha}{\gamma}t;\quad u'=\frac{\beta}{\gamma}u;\quad s=2(P_A\cdot P_B)$$

$$t=-2(P_A\cdot P_C);\quad u=-2(P_B\cdot P_C)$$

the limits of integration are as follows:

$$-\ln\frac{\gamma-x_1}{x_1}\le\Delta\le\frac{\gamma-x_2}{x_2}$$

It is obvious that the polarization will not be zero only in the case when the amplitude of the process $a+b\to c+d$ has both the imaginary and real parts. The usual assumption about the possibility of describing the process of parton scattering in the Born approximation results in a purely real amplitude and zero polarization. In the quark-gluon model the polarization is due to interference of one- and two-gluon exchanges and is very sensitive to the properties of the model. The calculations for the case when A is a nucleus differ from those when A is a particle by the fact that $Q_{A/a}$ contains the A dependent probability P_N of the type (2.13) or (2.14):

$$Q_{A/a}(\alpha)=\sum_{n=1}^{A}\left\{{n \atop A}\right\}q^{n-1}(1-q)^{A-n}Q_{nN/a}\left(\frac{\alpha}{n}\right). \tag{3.21}$$

We draw the following qualitative conclusions about the polarization properties:

a) polarization strongly depends on θ with a peak around 90°;

b) the product $k\cdot P_C$ weakly depends on the k value

$$x\approx\frac{E-p_z}{m_p}\approx k$$

(In our notation, $k=N^{min}$ is the cumulative number); P_C is the cumulative particle polarization;

c) for $E=\frac{s}{2m_p}\ge 5\div 10$ GeV polarization is energy independent;

d) polarization weakly depends on the beam and target.

The comparison of the measured polarization of Λ-particles produced in a process with large $p_\perp$[39)]

$$p+Be\to\Lambda+X \quad \text{at } p_p=300 \text{ GeV/c}$$

with that of cumulative Λ particles in processes

$$\pi+A\to\Lambda+X^{26,27)} \qquad \text{and} \qquad n+A\to\Lambda+X^{26)}$$

is in agreement with these conclusions. In the case of unpolarized particles (or particles with spin 0) eq. (3.20) reduces to eq. (2.3)

$$d\sigma=\sum_k P_k^A\, d\sigma_p\left(x_2,\frac{x_1}{k}\right)\left(1-\frac{x_1}{k}\right)^{\sigma(k-1)} \tag{3.22}$$

$x_1\approx N^{min}$; $d\sigma_p$ is the cross section of an one-nucleon process, the last factor is due to an excessive number of passive quarks (see also ref.[40)]). We estimated P_N in eq. (2.3) starting from the assumption on a uniform distribution of quarks over the nucleus. Essential deviations from the result of this estimation were obtained in P_N calculations[41)] on the basis of the quark bag model.

The study of multiquark fluctuations in nuclei, the so-called fluctons, is of much importance from the point of view of the study of quark dynamics. One of the interesting predictions of ref.[41)] consists in that for $N^{min}>4$ the cumulative effect probability must be negligible.

A fair number of papers is devoted to the investigation of the cumulative effect models on the basis of relativistic generalizations of the standard nuclear physics which do not take into account explicitly the quark structure of nucleons. The IMF apparatus is applied to the wave functions of nuclei consisting only of nucleons. The authors of ref.[42] consider that the applicability of relativistic quantum mechanical description of, e.g., the deuteron as a system of only two bodies is based on the absence of noticeable inelasticities in the NN scattering phase shifts up to relative motion momenta $\lesssim 1$ GeV/c. In ref.[42] many experimental facts on the cumulative production of mesons and protons are analysed on the basis of the hypothesis on nucleon pairing correlations in nuclei which is well known in nuclear physics. In particular, in the framework of this hypothesis, the one-particle distribution of cumulative protons in the process $h + A \to p + X$ is given by the formula

$$\frac{1}{\sigma_{tot}} E \frac{d\sigma}{d\vec{p}} = A \cdot \kappa \frac{\rho(M_{NN}^2)}{1-\beta}$$

Here $\rho(M_{NN}^2)$ is the probability density for a correlated nucleon pair. In the case of the deuteron it is normalized by the condition

$$\int \rho(M_{NN}^2) \frac{d^2 r \cdot d\beta}{\beta(1-\beta)} = 1$$

$$M_{NN}^2 = \frac{m^2 + r^2}{\beta(1-\beta)}$$

and for the nucleus at rest

$$\beta = \frac{\sqrt{m_p^2 + p^2} - p_z}{M_D}$$

(comp. eqs. (3.11) and (3.19)); κ takes into account a possible screening in the interaction of a hadron with a correlated pair; σ_{tot} is the total cross section for the interaction $h + A$.

The authors stress that it is just the quantity $\frac{1}{\sigma_{tot}} E \frac{d\sigma}{d\vec{p}}$, rather than $\frac{1}{\sigma_{in}} E \frac{d\sigma}{d\vec{p}}$, must weakly depend on the kind of the incident particle and criticize paper[28] in which the inverse assertion is given. Papers[42] contain also a criticism of the models of refs.[43,44].

An interesting rough version of the model defined by eq. (2.3) was suggested in ref.[45]. The simplification of eq. (2.3) consists in the following (let particle II be a parton

$$\rho_I^p(s, y, r) \approx \rho_p^p(\langle N_I \rangle s, y + \log \langle N_I \rangle, r) \tag{3.23}$$

where $\langle N_I \rangle \approx A_I^{1/3}$ is the effective number of nucleons involved in the collision. Dar and some other authors showed that eq. (2.23) is an economic means for describing a large amount of experimental material.

All the described models are of a tentative character and have a restricted predictive power. Especially, this concerns the quantum number dependence which is usually assumed to be weak. However, following the basic hypotheses, we are dealing with "large hadrons", "tubes", "fluctons", "quark plasmons", i.e., with objects possessing, e.g., large baryon charge and hypercharge. According to the ideas about gauge vector fields, the integral of motion must be exhibited dynamically (the coupling constant is proportional to the integrals of motion). In particular, it may be expected[51] that in just the same way as the coherent electromagnetic interactions with nuclei are proportional to the squared charge Z^2 the cross section of production of vector mesons ω and ϕ will be proportional, for example, to the squared hypercharge of a "quark plasmon" which emit these mesons. In ref.[51], in the framework of the two-component model, which takes into account "noncoherent" (parton) mechanism and "coherent" mechanism of Regge exchange one gives a numerical estimate of growth of the meson yield compared to the η meson yield in the cumulative region. This growth is due to a possible proportionality of the amplitude of fragmentation of a baryon cluster ("plasmon") with meson emission to the hypercharge and baryon number of the cluster. The check of this prediction in experiments on nuclear limiting fragmentation

$$A \to \omega \mid_{\to \pi^o \gamma} \qquad \text{and} \qquad A \to \eta \mid_{\to \gamma\gamma}$$

is of much interest from the point of view of both the verification of the important dynamic symmetry and clarification of the role of the multiquark states in cumulative particle production.

IV. Present Status and Perspectives of the Studies in the Field of Relativistic Nuclear Physics at the Joint Institute for Nuclear Research

For the nearest years the main tools of relativistic nuclear physics will be proton synchrotron and detectors of elementary particle physics. The intensity of the extracted nuclear beams is already now much higher than the intensity of the beams of secondary particles (pions, kaons etc.) which the existing detectors are rated at. The secondary beams, even pion ones, with an intensity 10^5 - 10^6 part./sec are considered to be good for the available facilities and the extracted beams of relativistic nuclei have an intensity of 10^6 - 10^{11}part./sec. This provides us with an encouraging perspective of applying the existing detectors created for the work with secondary beams to relativistic nuclear physics. The relativistic nuclear beams, the parameters of which will be improved undoubtedly in the nearest future and the available detectors will make it possible to resolve many of the problems discussed.

Five electronic installations and three track detectors, liquid hydrogen bubble chamber, two-meter propane bubble chamber and two-meter streamer chamber, of the Dubna synchrophasotron are used for studies in the field of relativistic nuclear physics. For the exception of the Stavinsky team's installation, which was used to obtain the main results on the cumulative effect, all these facilities were intended for performing studies in the field of particle physics, and it is only lately that they have been adjusted for investigations with relativistic nuclei.

In the first installation of the Stavinsky's team specially created for studying processes of the type $p + A \to \pi(180^o)$ pions were detected by a DISC-type Čerenkov differential counter with a velocity resolution $\Delta\beta = \pm 3 \cdot 10^{-2}$ in the velocity range $0.7 \div 1.0$. The second version of this installation is a rotating magnetic spectrometer which allowed to perform detailed measurements of the angular cumulative particle distributions. Events were there extracted by an independent measurement of the time of flight on two bases (4 and 1 meter) with an accuracy 150÷200 psec and measurement of ionization losses and intensity of the Čerenkov burst in a solid radiator. The description of these facilities is given in refs.[46) and[47). Among the recent results of the Stavinsky's team it is necessary to mention a detailed study of the limiting fragmentation of light nuclei. The measurements are led to particle energies corresponding to the kinematic limit of interaction with the nucleus as a whole. The investigation of elastic pd scattering with large momentum transfers has been carried out.

The study of the nucleus-nucleus scattering at small momentum transfers is performed on the internal accelerator targets[48). This technique which was developed on the Dubna Synchrophasotron with the participation of the same physicists started the well-known investigations with the aid of a supersonic jet target on the Serpukhov and Batavia accelerators. This same technique was lately employed to perform a search for the isotope ^{10}He in relativistic nuclear collisions.

Two one-arm magnetic spectrometers with proportional chambers are used for measuring the inclusive cross sections for relativistic nuclear collisions[49,50). An installation "Photon" is oriented to studying relativistic nuclear collisions with emission of neutral particles (π^o , η^o , ω^o). It is a 90 channel Čerenkov hodoscope of lead glass in which the gamma quantum energy is measured. The direction of gamma quanta is measured by 32 spark chambers with a magnetostrictive readout. The accuracy of measurement of the gamma quanta direction depends on the thickness of the converters and amounts to 3.4 mrad. A large complex of electronic apparatus and a on-line computer of the installation "Photon" make it possible to study effectively multiple photon emission in relativistic nuclear collisions, in particular, the problem formulated in ref.[51] on cumulative production of vector mesons.

Among track devices the 2-meter propane chamber has been advanced most greatly for the purposes of relativistic nuclear physics. A typical photograph of this chamber is given in fig. 19 where collisions of carbon nucleus of a momentum of 50 GeV/c with a tantalum nucleus (vertical lines are tantalum plates in the working volume of the chamber) and a carbon nucleus of propane are shown. The multiple particle production in relativistic nuclear collisions is found to be simpler from the topological viewpoint than that in p−p collisions at an energy of hundreds of GeV. A large group of physicists under the leadership of Soloviev has solved the principal problems of handling of such photographs and has obtained a large amount of experimental information on multiple production processes of relativistic nuclear physics. The characteristics of multiple particle production defined in Section I have been obtained. The extraction of multinucleon interactions is made by the number of proton spectators Z_s :

$$\langle \nu_p \rangle = 2(Z - \langle Z_s \rangle)$$

where $\langle \nu_p \rangle$ is the average number of the nucleons of a bombarding nucleus which have interacted with a target, Z the charge of the bombarding nucleus, $\langle Z_s \rangle$ the average charge of stripped particles.

The experimental data show that the average number of nucleons taking part in the interaction is rather large. For example, it reaches $\langle \nu_p \rangle = 6.00 \pm 0.60$ for the collisions of the carbon nucleus with the tantalum nucleus. The table gives the values of the ratio $\langle n_- \rangle / \langle \nu_p \rangle$ or the average numbers of the produced negative particles per nucleon of an incident nucleus.

bombarding nucleus \ target	C_3H_8	C	Ta
P	0.37 ±0.04	0.40 ±0.04	0.55 ± 0.03
d	0.37 ±0.03	0.39 ±0.04	0.55 ±0.05
He	0.37 ±0.03	0.39 ± 0.04	0.50 ±0.03
C	-	-	0.52 ±0.06

The constancy of the values tabulated testifies in favour of the definition of $\langle \nu_p \rangle$ and the fact that the nucleons of colliding nuclei interact, on the average, in an independent manner.

Of a particular interest are the search for and study of multibaryon resonances the existence of which is predicted by the quark bag theory. The (Λp) and possibly, ($\Lambda\Lambda$) and ($\Lambda\Lambda p$)resonances discovered by Shakhbasian on the basis of the study of photographs from the propane bubble chamber were lately interpreted[52-55] as multiquark formations in a single "bag". The confirmation of the existence of such large "quark plasmons" would be very important, in particular, it would mean that we have already discovered metastable states of superdense nuclear metter, i.e., multibaryon states possessing elementary particle density. In the same experiments it is found to be possible to study the cumulative production of Λ particles,

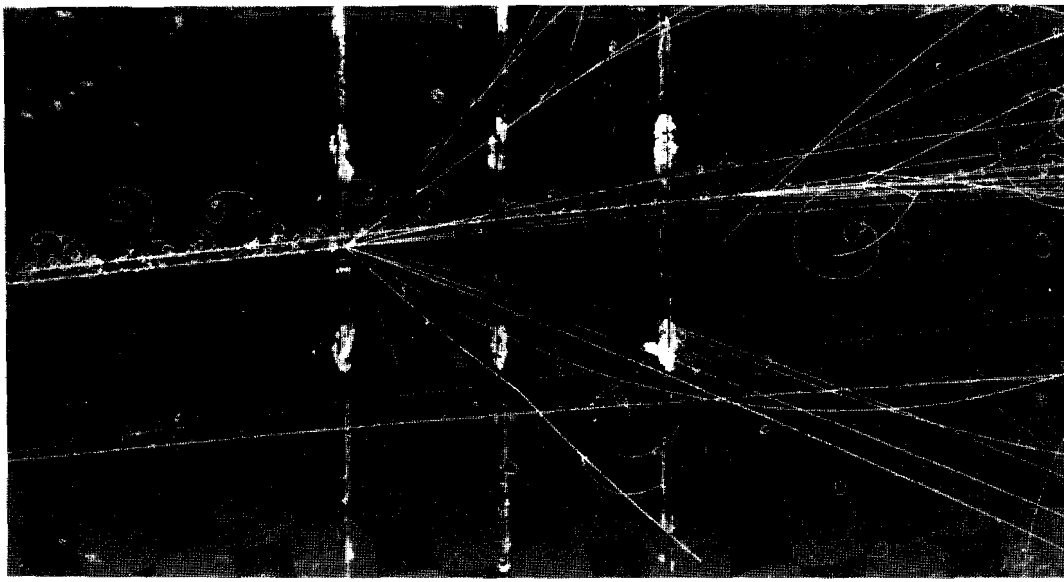

Fig. 19

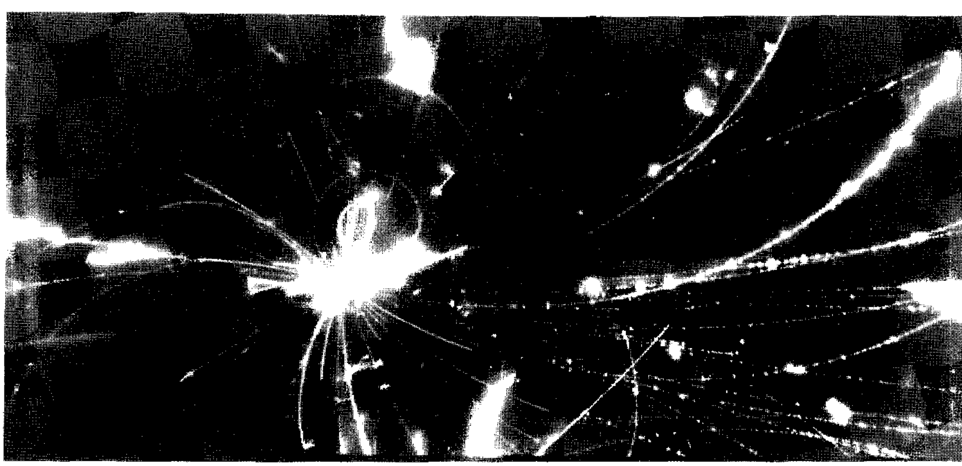

Fig. 20

including the study of their polarization (see above). The relationship between the cumulative effect and the manifestation of quark plasmons is one of the most interesting and important objects of the investigations in relativistic nuclear physics.

Fig.20 presents a photograph of the interaction of the ^{16}O nucleus with a momentum 72 GeV/c with a target in the 2-meter streamer chamber. The description of this apparatus is given in ref.[54]). This chamber was used to show the analogy of the multiplicity distributions of negative particles in nucleus-nucleus collisions and in p-p collisions at high energies.

The first experiments on deuteron acceleration on the Dubna Synchrophasotron in 1970 showed that in order to proceed to the acceleration of nuclei with the aid of ordinary proton accelerators the accelerating system needs not be strongly modified. Thus, any high energy accelerator can be adapted to accelerate deuterons and α particles.

In order to pass to the acceleration of nuclei with large atomic masses a number of technological problems should be resolved. The main of them is obtaining of bare nuclei. The acceleration of partially ionized atoms imposes very strict requirements on the vacuum inside the accelerator chamber. To obtain bare nuclei it is suggested to create pre-accelerators with an intermediate stripping.

The Laboratory of High Energy Physics of the JINR started developing essentially new sources of multicharged ions: electron beam ion and laser sources. The electron beam ion source invented by Donetz is a rather compact device which has reliably been running on our Synchrophasotron for a long time under operating conditions. The principle of operation of the source consists in the following. A certain amount of one-charged ions of an element to be accelerated is introduced into an electron beam of high density (hundreds of amperes per cm^2). The ions perform radial oscillations under the action of the forces of the electric field of the electron space charge. The interaction of the ions with the fast electrons of the beam produces a multiple ionization; the ion charge increases. The electron beam is placed in a deep vacuum in a strong longitudinal magnetic field (superconducting solenoid). Cryogenics makes it possible to obtain a magnetic field of practically any necessary value and reach a vacuum in an ionization region of 10^{-11} torr. This source is sometimes called "CREBIS" (cryogenic electron beam ion source). The present status of the relative investigations enables up to hope to obtain bare nuclei of an intensity of about $10^{11}\cdot 1/Z$.

The beam intensities of the Dubna Synchrophasotron which were used by the physicists in 1978 are tabulated.

Accelerated particles	Energy in GeV per nucleon	Particle intensity per pulse
protons	10	4.10^{12}
deuterons	5	5.10^{11}
^{4}He	5	3.10^{10}
^{12}C	5	2.10^{6}
$^{14}N, ^{16}O$	5	10^{4}
^{20}Ne	5	sufficient for experiments to be performed in the streamer chamber

In 1979-1980 an increase of the intensities of ^{12}C , ^{14}N , ^{16}O and ^{20}Ne is expected at the expense of an improvement of the injection system, but this increase will not be larger than by a factor of 10^2. An essential expansion of the beam facilities will take place after the creation of a synchrophasotron buster.

Relativistic acceleration of heavy nuclei and even intermediate mass nuclei requires creation of special injection complexes, pre-accelerators at an energy about 500 MeV/N, which are of a great value by itself. The high voltage injection resolves also vacuum problems in the main ring since for ions of an energy higher than 500 MeV/N the electron pick-up is nonessential even for rather moderate requirements on the vacuum.

Thus, the following research programme is worked out at the Laboratory of High Energy Physics of the JINR. During the nearest 4-5 years it is planned to use extensively the beams of relativistic nuclei of the Synchrophasotron up to 5 GeV/N energies. As we have already stressed, the limiting fragmentation of nuclei begins at an ion energy higher than 3 GeV/N. This ion energy range has as yet been obtained in no other accelerator centers. The available detectors will make it possible to realize a rather wide programme of investigations. By the present time it has been completed the construction of a large experimental hall of an area of about 10^3 m^2 in which a large number of simultaneously operating installations can be arranged on the extracted beams of the Synchrophasotron.

Further perspectives of our Laboratory are connected with the creation of a superconducting specialized accelerator of nuclei which will replace the Synchrophasotron. Some progress has also been made in the creation of superconducting magnets for accelerators (see, e.g. ref.[55)]). Some preliminary suggestions concerning the design of a superconducting accelerator of relativistic nuclei which was given the name "Nuclotron" are presented in refs.[56,57)]. They have underlain projects of construction of an injection complex of the Laboratory of High Energy Physics and, in particular, its intermediate ring accelerator the main parameters and the operating regime of which are given in ref.[58)]. The creation of the Nuclotron on the basis of the resources available at the Laboratory (buildings, tunnel, energetics, large experimental hall equipped with a system of channels, detectors, etc.) will make it possible to lower noticeably the cost of the accelerator complex.

The first stage of construction of the Nuclotron is the creation of a buster of an energy of few handreds of MeV/N. The use of it as a Synchrophasotron injector at this stage will essentially improve the JINR beam facilities. The buster beams will also be applied to studies of supersonic and high-temperature nuclear reactions, as well as to medical and space research, etc.

The creation of the intermediate ring accelerator-buster has attracted the attention of physicists of the Kurchatov Institute. They proposed an interesting research programme in the energy range up to 0.5 GeV/N.

Efforts of both the institutes were combined and the initial design of the intermediate accelerator[58)] underwent some changes[59)]. Other institutes of the JINR member-countries have expressed their interest in supporting the creation and development of the heavy-ion acceleration complex.

Thus, the Laboratory of High Energy Physics research programme implies constant development of the accelerator complex with actually unceasing and intense use of relativistic nuclear beams. This prodives us with large possibilities of performing investigation in the new and very perspective field of physics, relativistic nuclear physics.

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