On existence of a mixed phase and critical point

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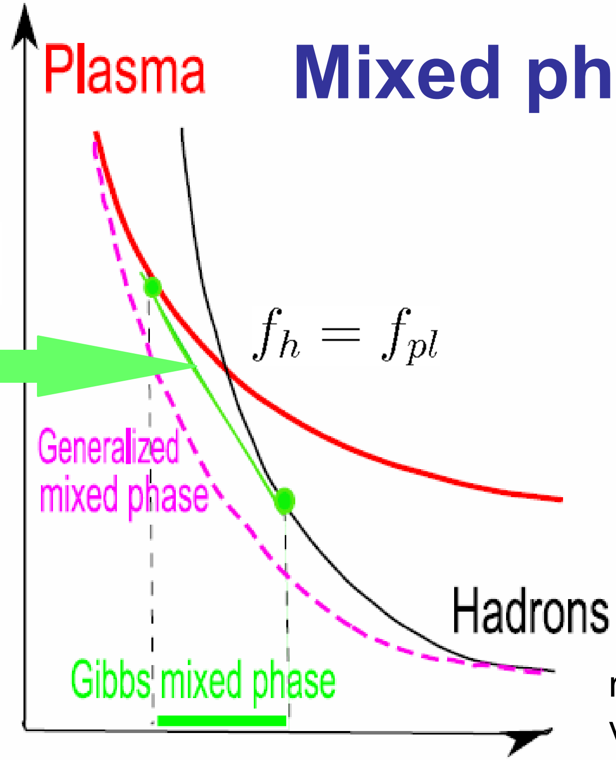
International meeting dedicated to 100-th anniversary of *V.I.Veksler's* birthday and 50-th anniversary of Synchrophasotron starting up, Dubna, October 10-12, 2007

Mixed phase concept

reduced
free energy

$$f = \frac{F}{N} = -\frac{T \ln \mathcal{Z}}{N}$$

Maxwell construction



reduced
volume

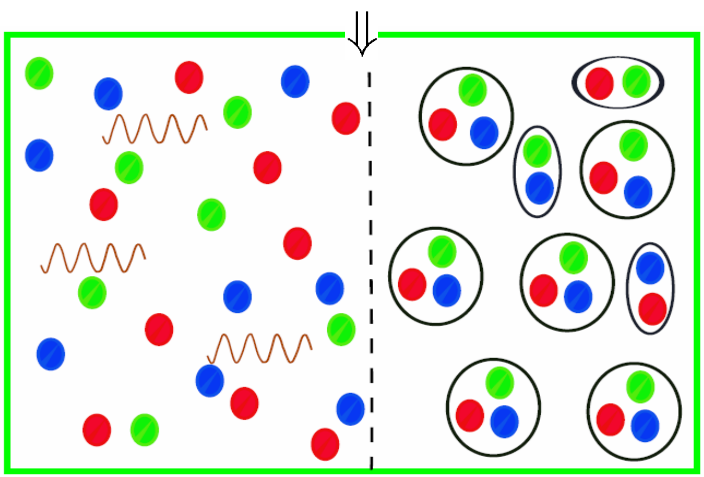
$$v = \frac{V}{N} = 1/\rho$$

$$(\frac{\partial P}{\partial V})_{N,T} < 0$$

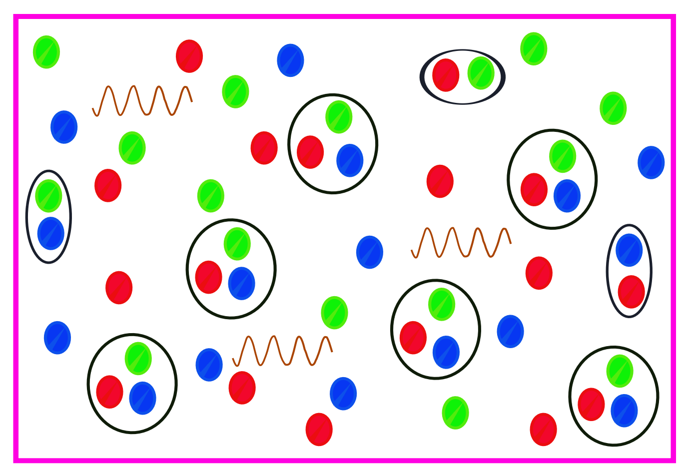
$$P = -(\frac{\partial F}{\partial V})_{N,T}$$

$$\frac{\partial^2 F}{\partial V^2} > 0$$

The Gibbs mixed phase (spatially separated)



Generalized mixed phase (homogeneous)



Deconfinement transition (an illustrative example)

for the massless Stefan-Boltzmann gas

$$P = g_i \left(\frac{\pi^2}{90}\right) T^4$$

$$\varepsilon = g_i \left(\frac{\pi^2}{30}\right) T^4 = 3P$$

g_i – degeneracy factor

$$c_s^2 = P/\varepsilon = 1/3$$

Two-phase model: pions ⟺ quarks+gluons (pl)

Pion gas

$$g_\pi = 3$$

$$P_\pi = 3\left(\frac{\pi^2}{90}\right) T^4$$

$$\varepsilon_\pi = 3\left(\frac{\pi^2}{90}\right) T^4 = 3P_\pi$$

Plasma

$N_f=2$, $N_c=3$

$$g_{pl} = 2\left(\frac{N_c N_f}{4} + N_c^2 - 1\right)$$

$$P_{pl} = 37\left(\frac{\pi^2}{90}\right) T^4 \boxed{- B}$$

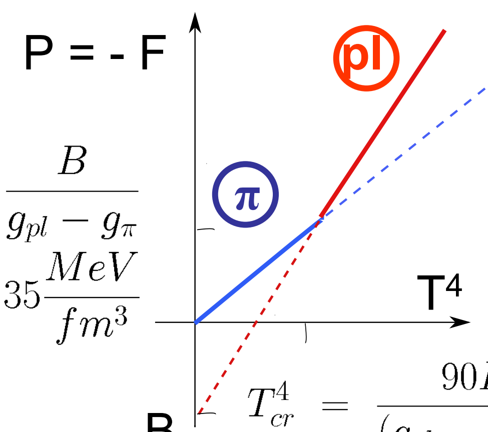
$$\varepsilon_{pl} = 37\left(\frac{\pi^2}{30}\right) T^4 \boxed{+ B} = 3P_{pl} + 4B$$

Gibbs conditions fot the first order phase transition

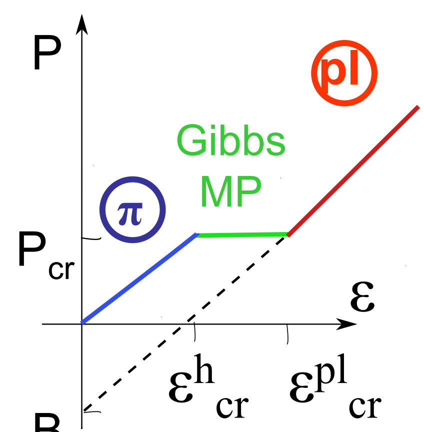
$$P_\pi = P_{pl} = P_{cr}$$

$$T_\pi = T_{pl} = T_{cr}$$

$$P_{cr} = \frac{B}{g_{pl} - g_\pi} \approx 35 \frac{MeV}{fm^3}$$



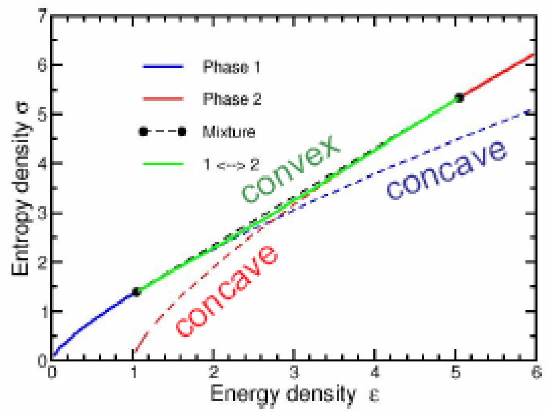
$$T_{cr}^4 = \frac{90B}{(g_{pl} - g_\pi)\pi^2} \approx (170\ MeV)^4$$



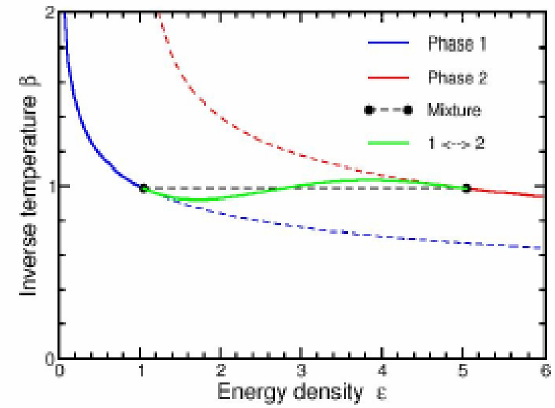
Latent heat ~1.5 GeV/fm^3

Simplest example (no conserved charge)

Entropy density: $\sigma(\varepsilon)$

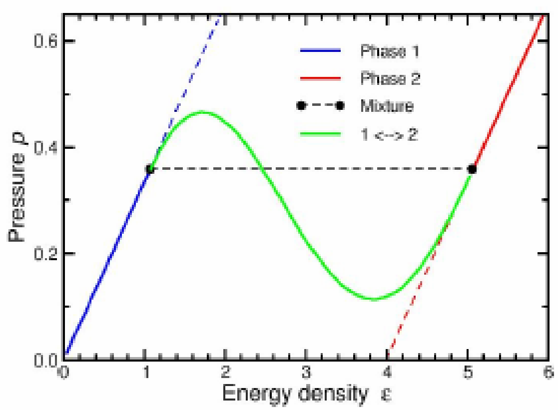


Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$

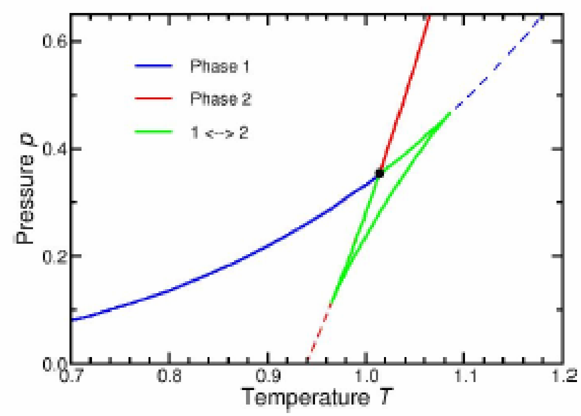


Equation of State

Pressure: $p(\varepsilon) = T\sigma - \varepsilon$



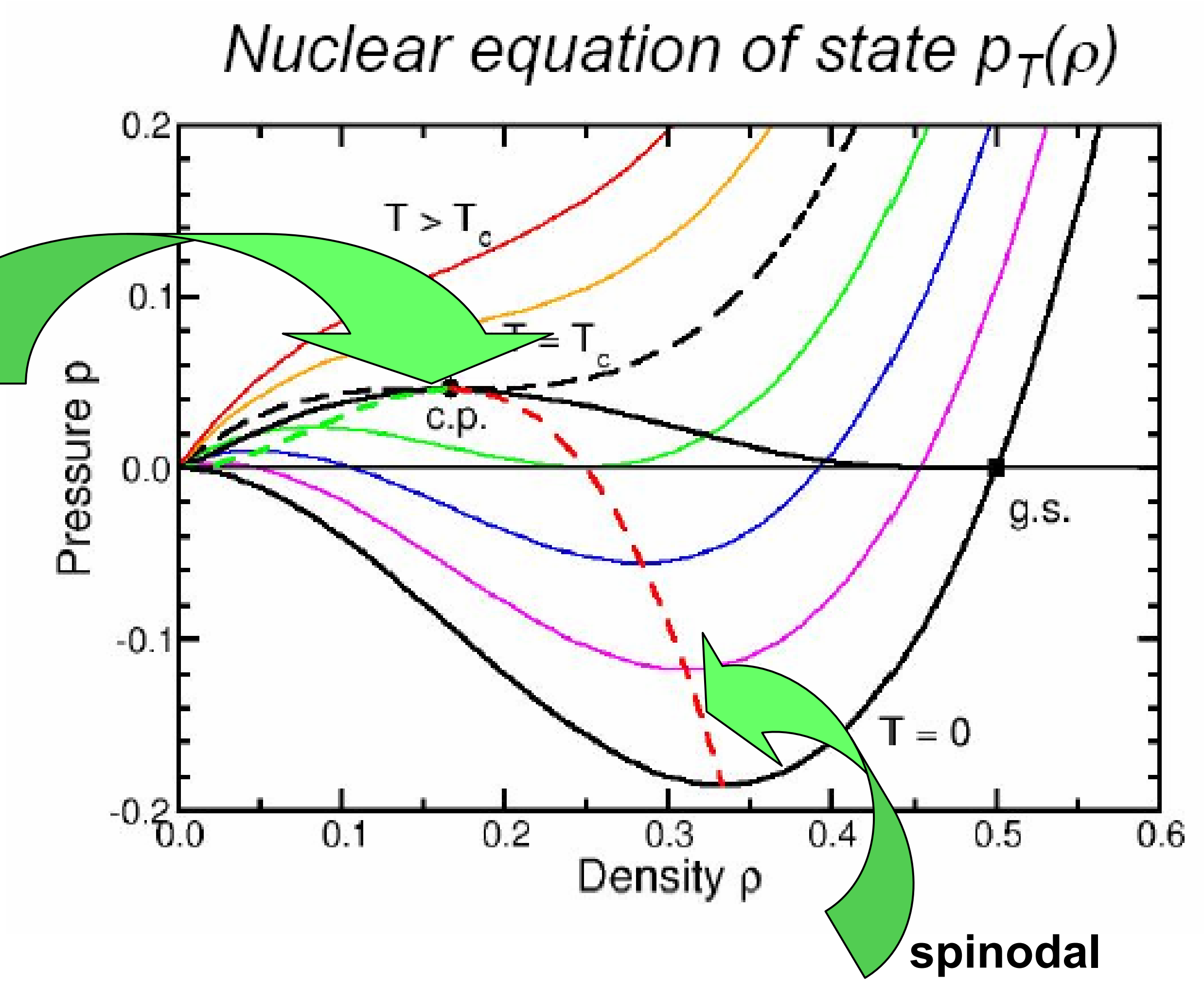
Pressure: $p(T)$



J.Randrup, Nucl.Phys. A752 (2005) 384c

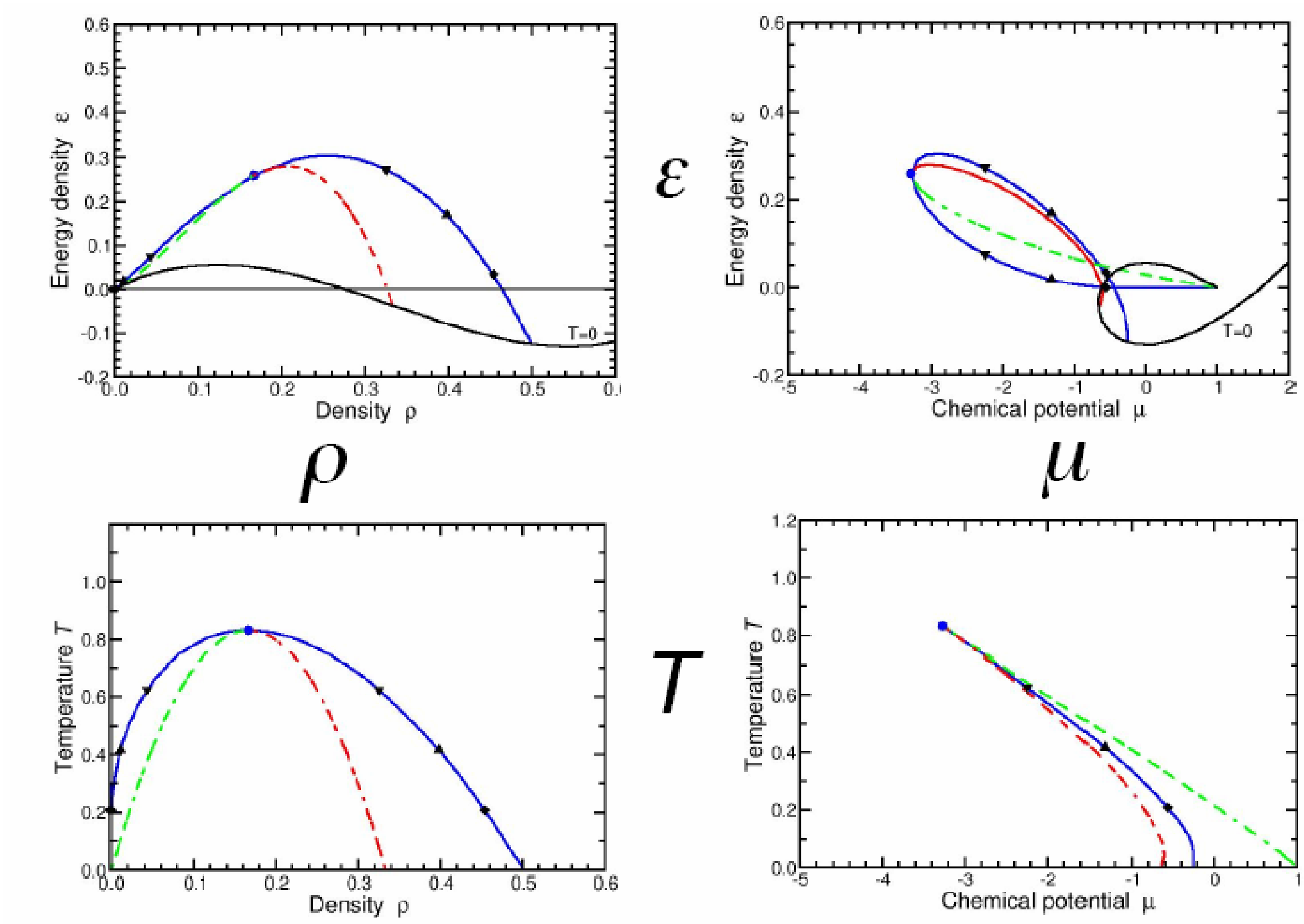
Familiar example (one conserved charge)

Critical end point



spinodal

Nuclear phase diagram in different representation (one conserved charge)

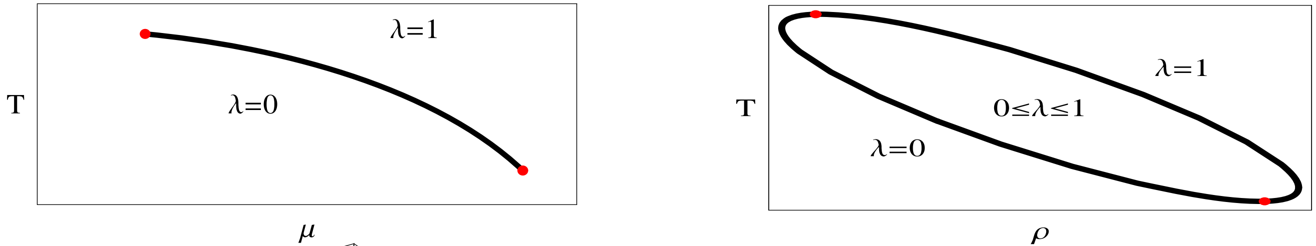


J.Randrup, CEOD, Darmstadt, July 9-13, 2007

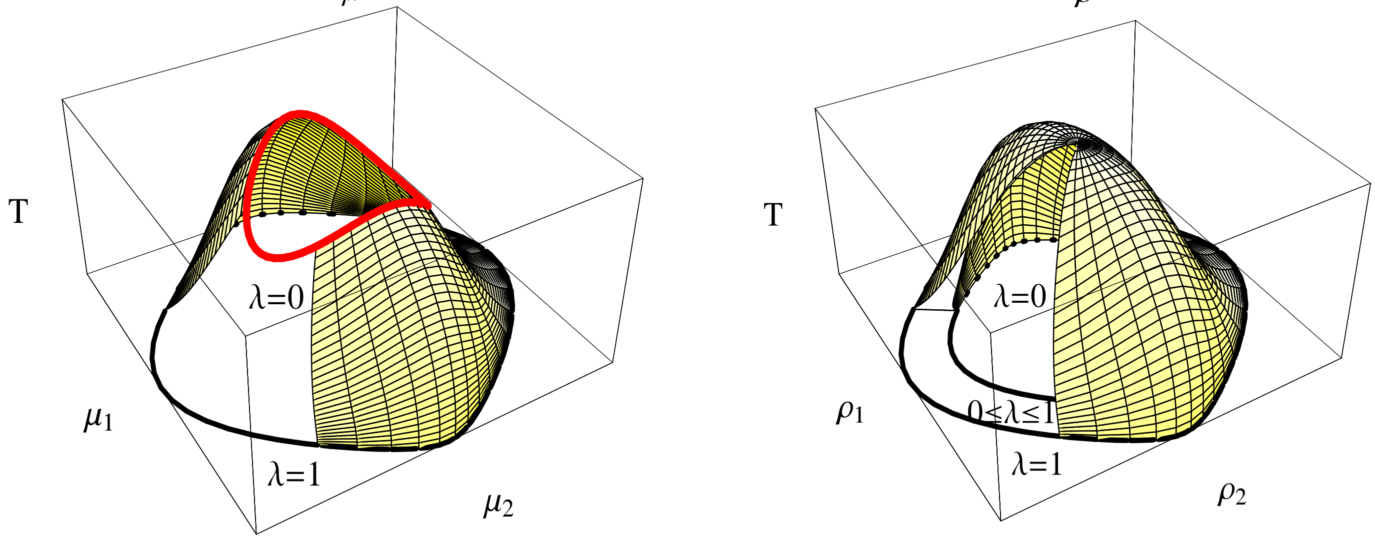
Two conserved charges

for an extensive thermodynamic quantity $A = \lambda\, A_2 + (1-\lambda)\, A_1$ $\lambda = V_2 / V$

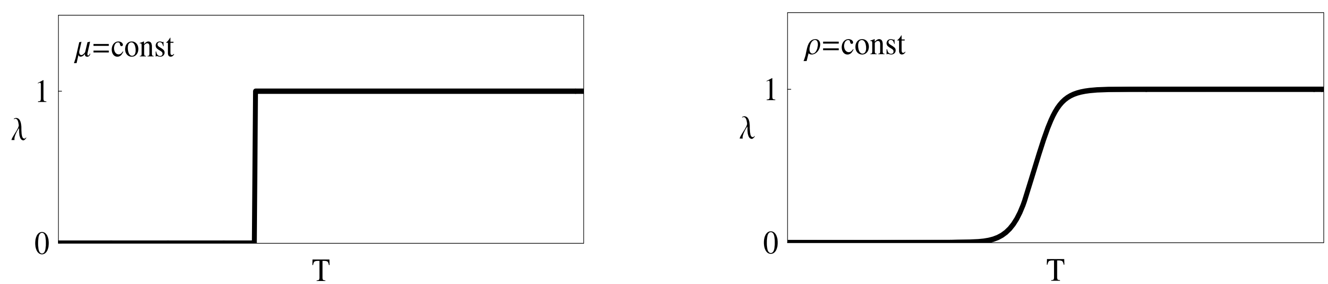
1 charge ⇨



2 charges ⇨



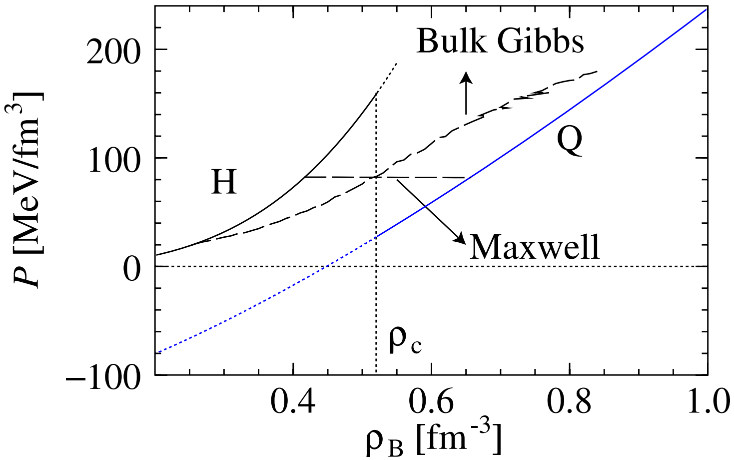
volume fraction ⇨

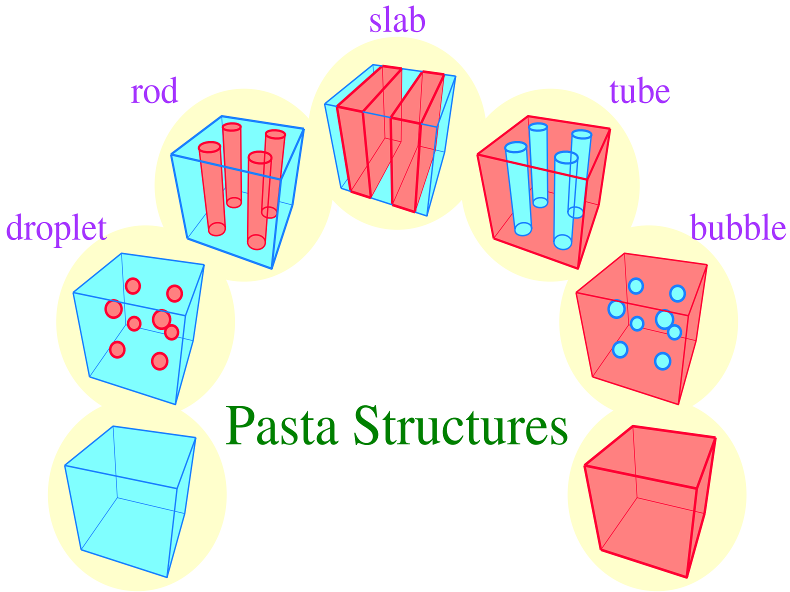


A.Sissakian, A.Sorin, V.Toneev, nucl-th/0608032

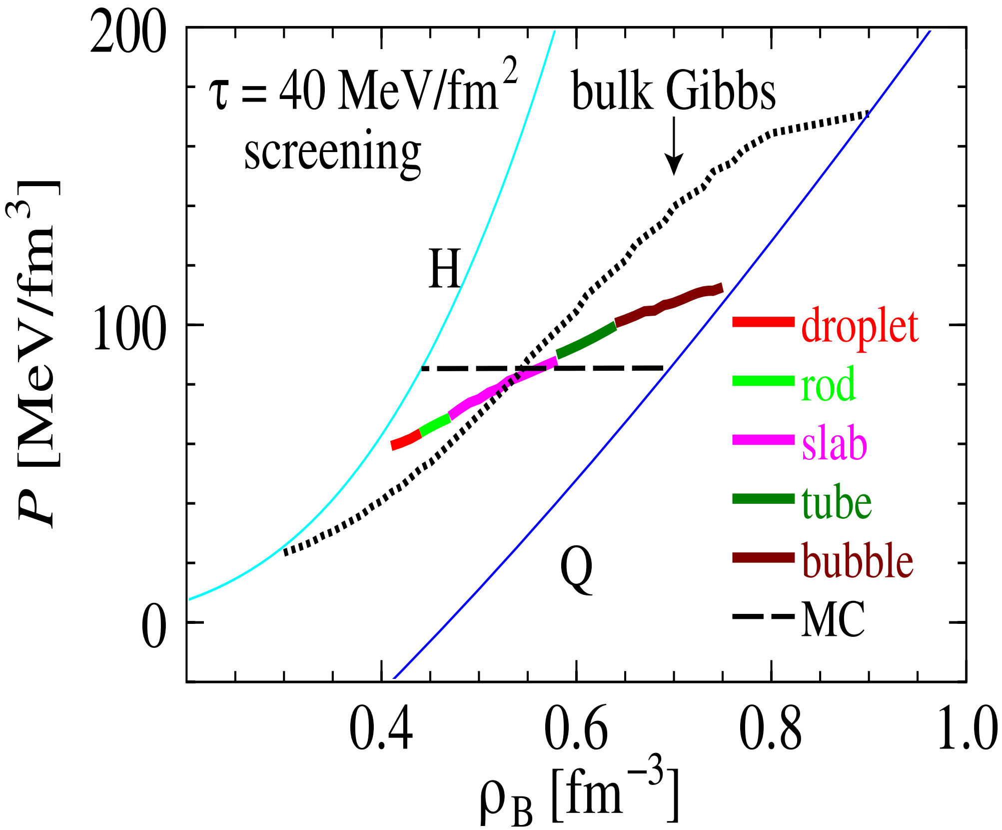
“Pasta” structures (finite size effects)

Compact stars, T=0





T.Maruyama et al., nucl-th/0605075



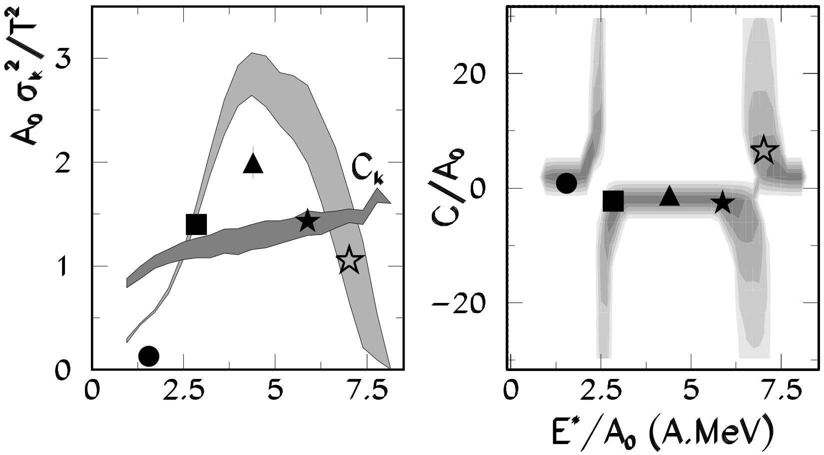
Charge density and particle density are non-uniform

Heat capacity (nuclear matter)

$$T^{-1}=\frac{\partial S}{\partial E} \qquad C_i^{-1}=-T^2\frac{\partial^2 S}{\partial E^2}$$

$$E_t=E_1+E_2 \qquad T_1=T_2=T$$

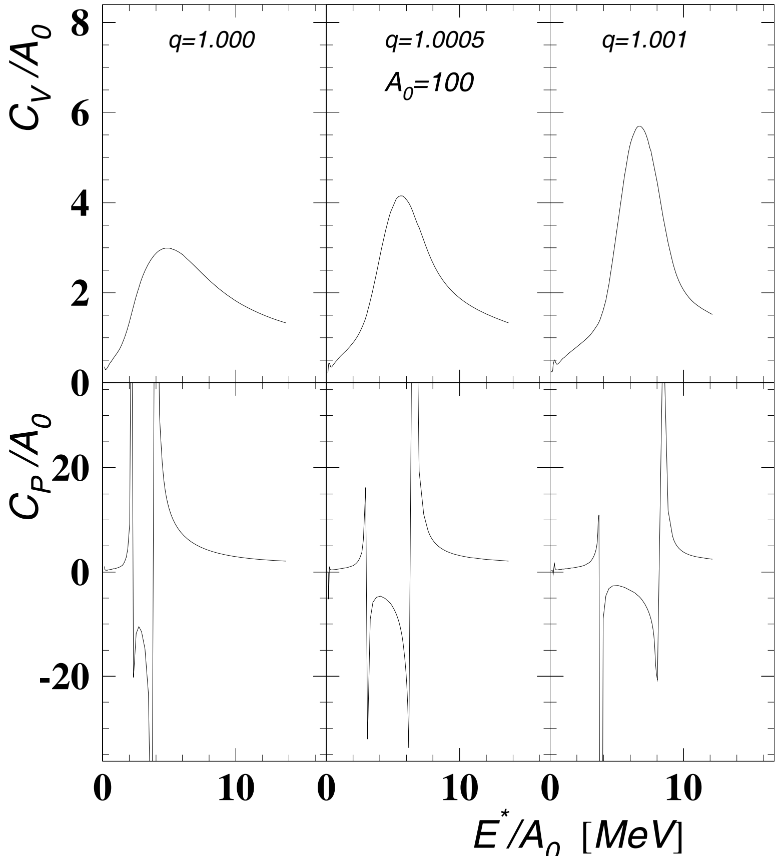
$$C_t\approx C_1+C_2=\frac{C_1^2}{C_1-\sigma_1^2/T^2}$$



heat capacity is negative in the mixed phase

M.D'Agostino et al. NP **A734** (2004) 512

Statistical Tsallis model of multifragmentation



K.Gudima, A.Parvan, M.Ploszajczak V.Toneev, PRL **85** (2000) 4691

Susceptibility

Nambu--Jona-Lasinio model

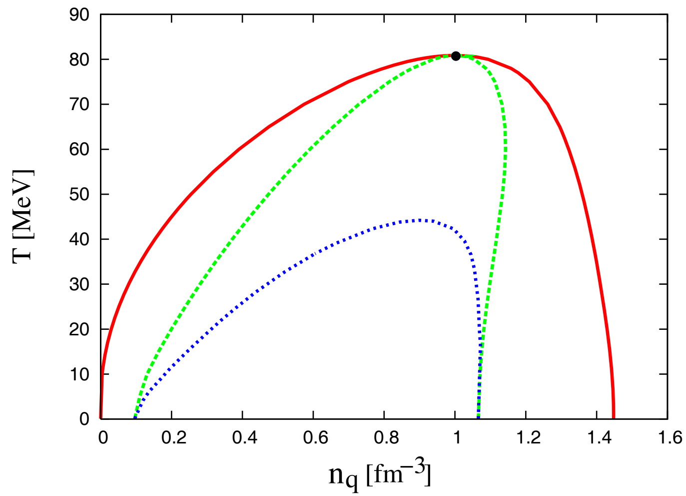
$$\chi_q = -\frac{1}{V}\frac{\partial^2 F}{\partial \mu_q^2} = \frac{1}{V}(< N^2 > - < N >^2)$$

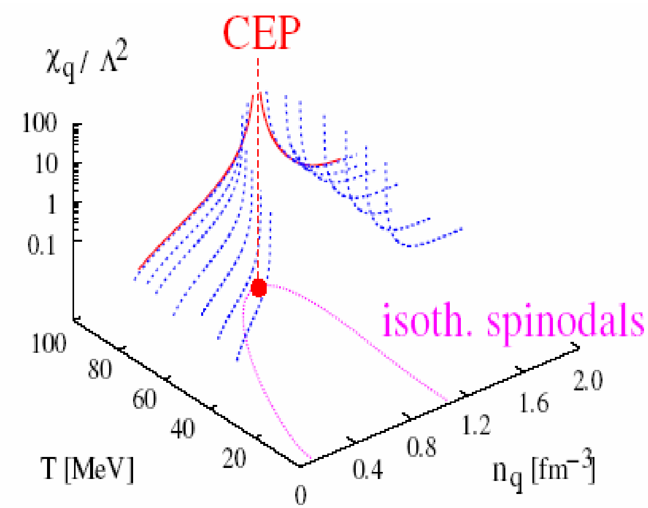
$$\left.\frac{\partial P}{\partial V}\right|_T = 0 \quad : \text{ isothermal}$$

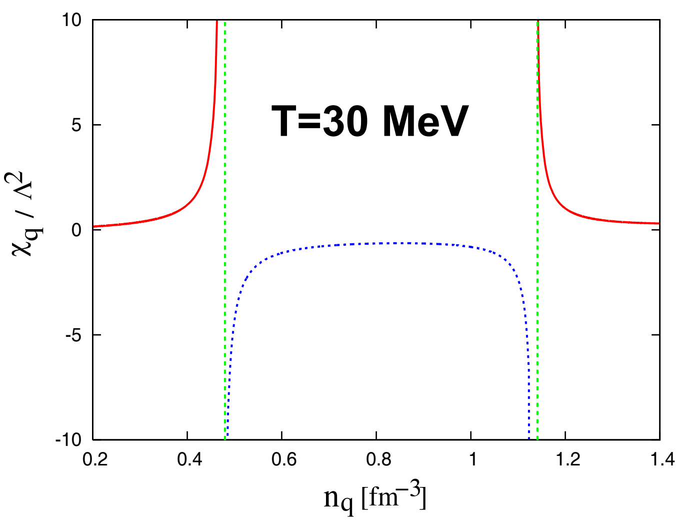
$$\left.\frac{\partial P}{\partial V}\right|_S = 0 \quad : \text{ isentropic}$$

T_c=81 MeV

μ=330 MeV







poles, negative branch

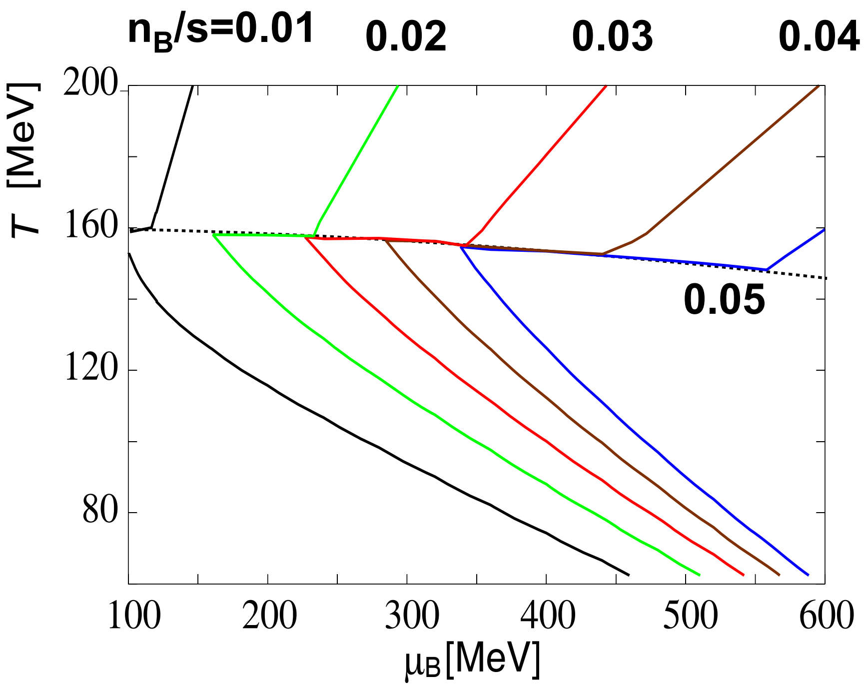
instability region shrinks toward the critical end point

K.Redlich, B.Friman, C.Sasaki,

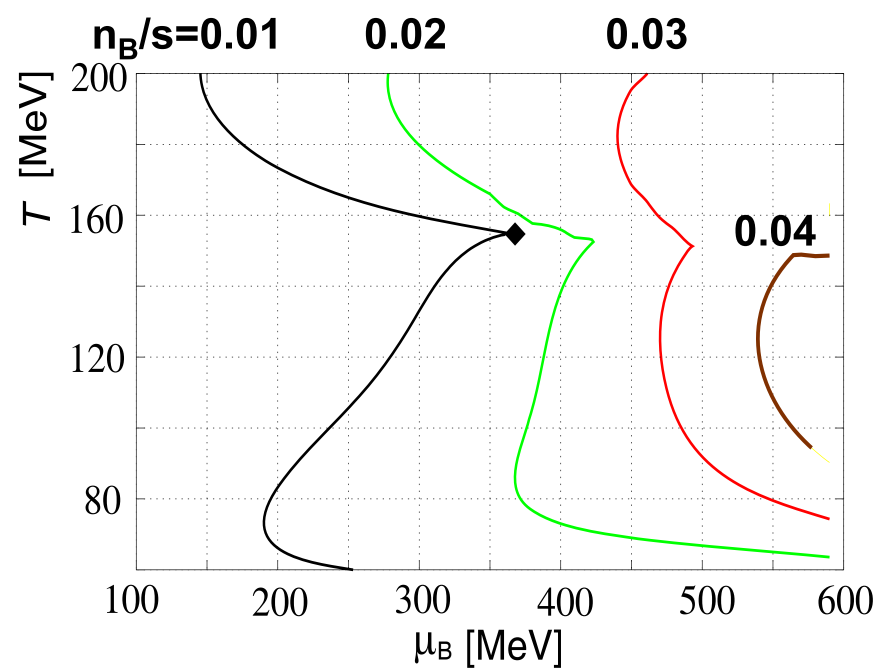
J. Phys. **G34** (2007) 437

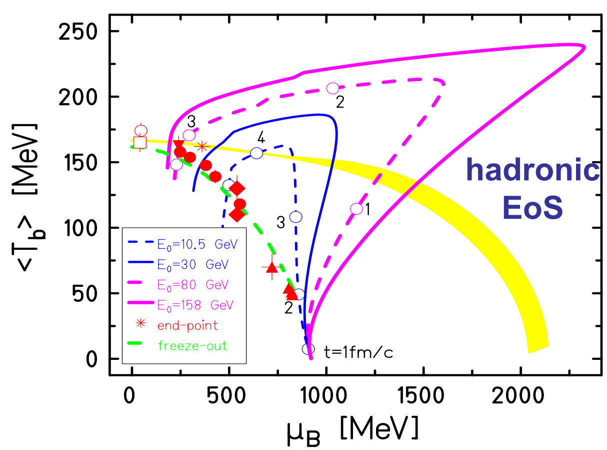
Time evolution near the QCD critical point

Two-phase EoS (first order)



Two-phase EoS with CP



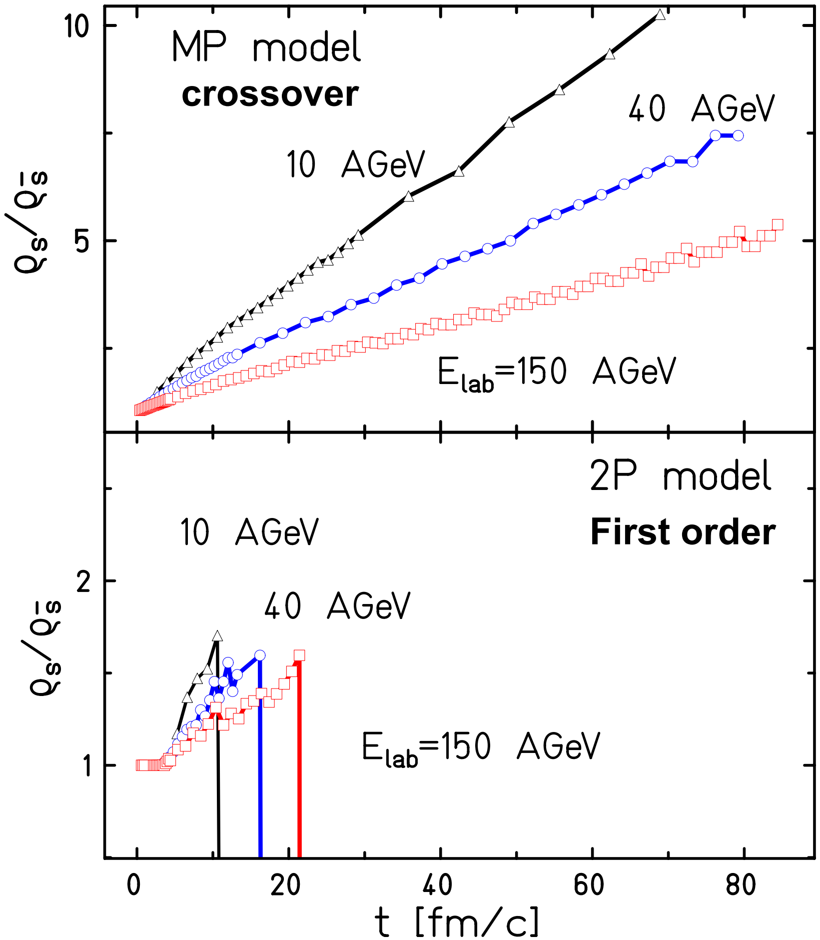


Critical point acts as an attracter of isentropic trajectories

C.Nonaka, M.Asakawa,
Phys. Rev. **C71** (2005) 044904

Distillation in the mixed phase

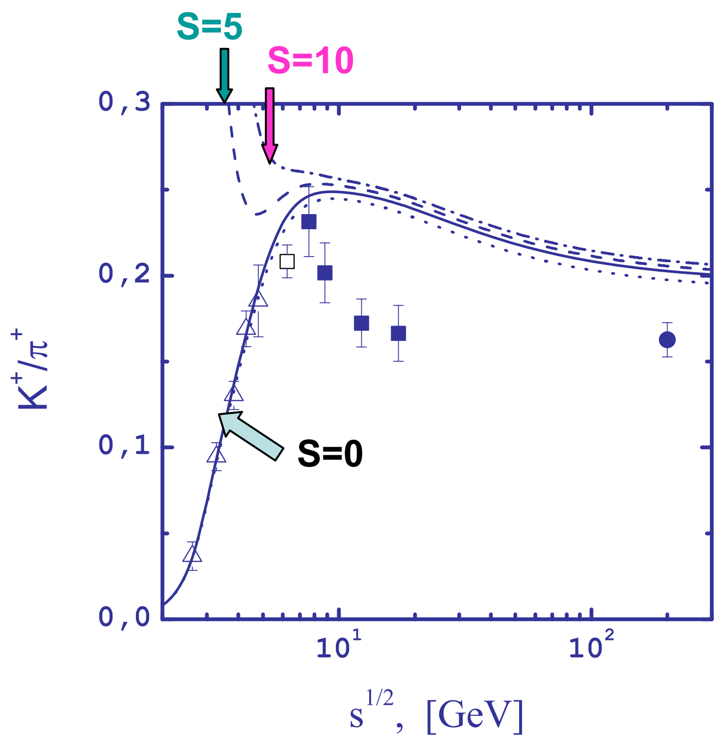
for unbound quarks in a fireball



strangeness separation

V.Toneev et al., Eur. Phys. J. **C32** (2004) 415.

K^+/π^+ excitation function, statistical equilibrium along the freeze-out line

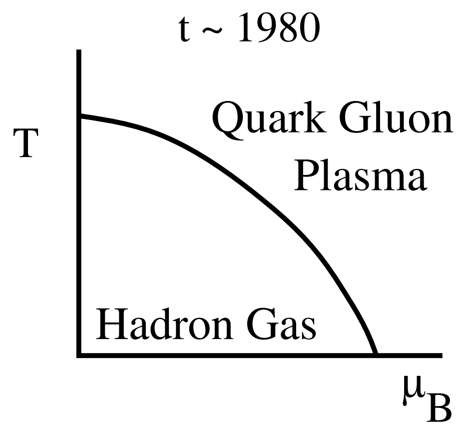


At CP the K^+/π^+ ratio should jump down on the S=0 curve

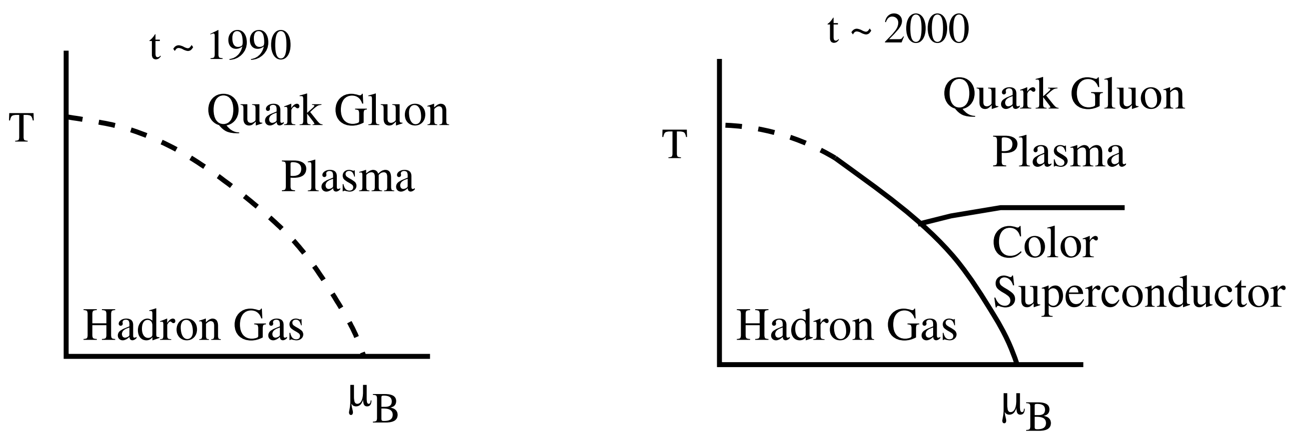
V.Toneev, A.Parvan, J. Phys. **G**: Nucl. Part. Phys. **31** (2005) 583

Lattice QCD calculations (history)

The Evolving QCD Phase Transition



Critical Temperature 150 - 200 MeV ($\mu_B = 0$)
Critical Density 1/2-2 Baryons/Fm^3 ($T = 0$)

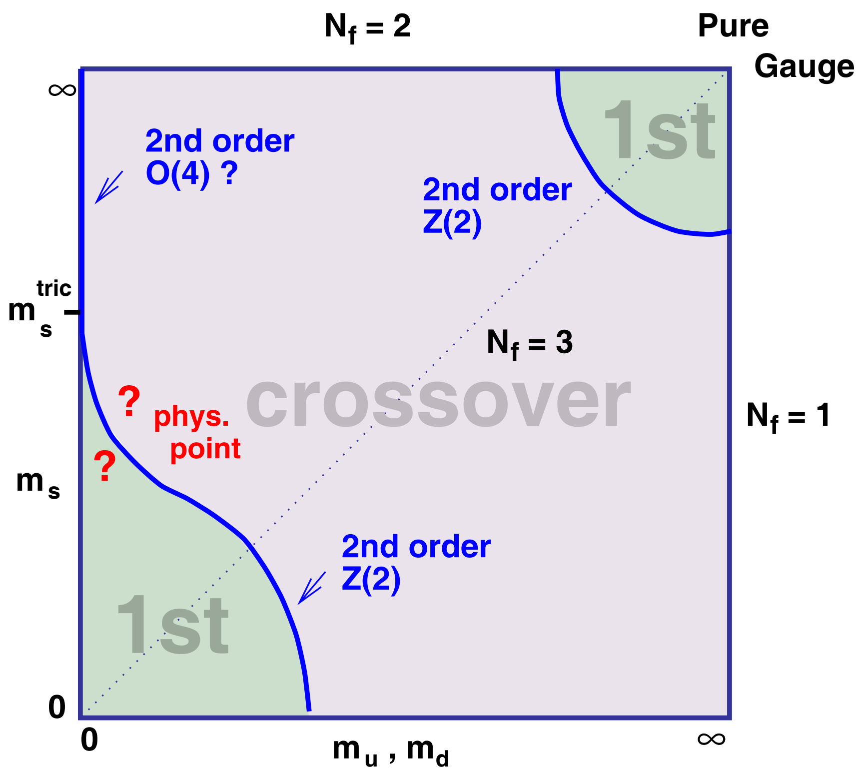


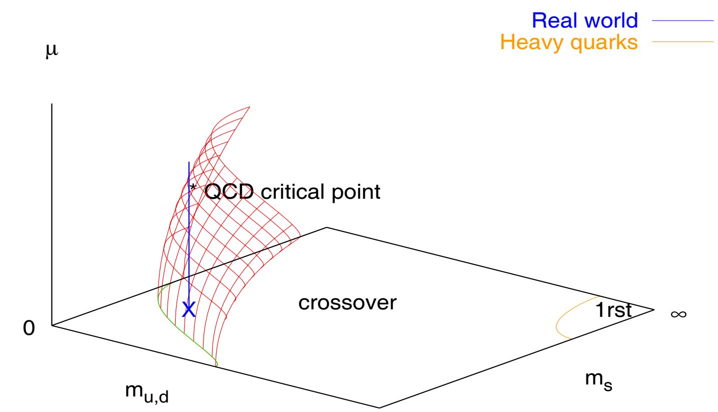
L. McLerran, Prama **60** (2003) 575

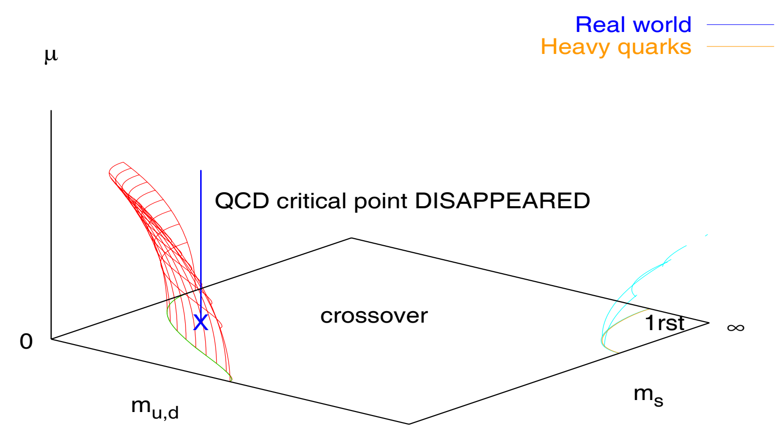
Lattice QCD predictions for the order of PT

E.Laermann, O.Phillipsen,
Ann.Rev.Nucl.Part. Sci.,51 (2003) 163

S.Kim et al. hep-lat/510069





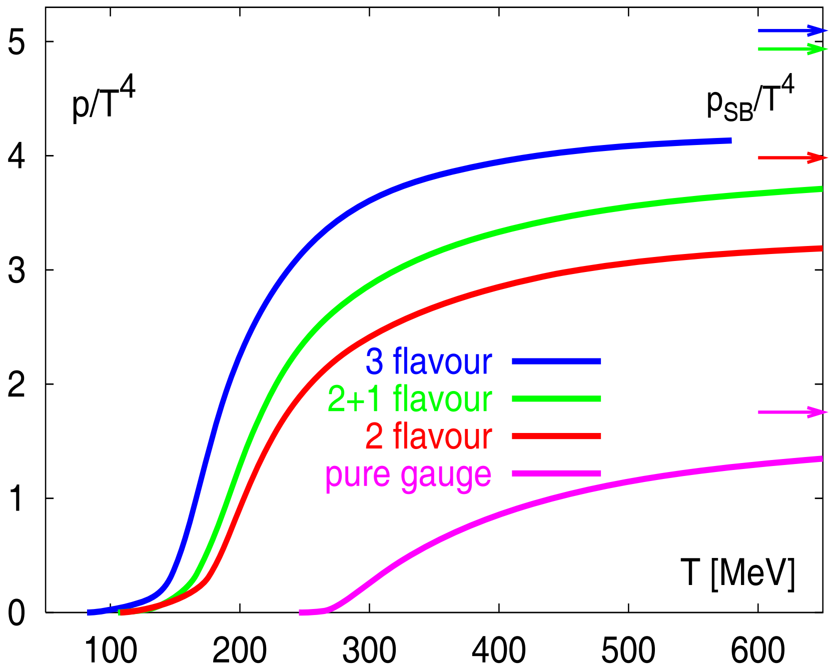


$\mu_B=0$

$\mu_B \neq 0$

Lattice QCD thermodynamics

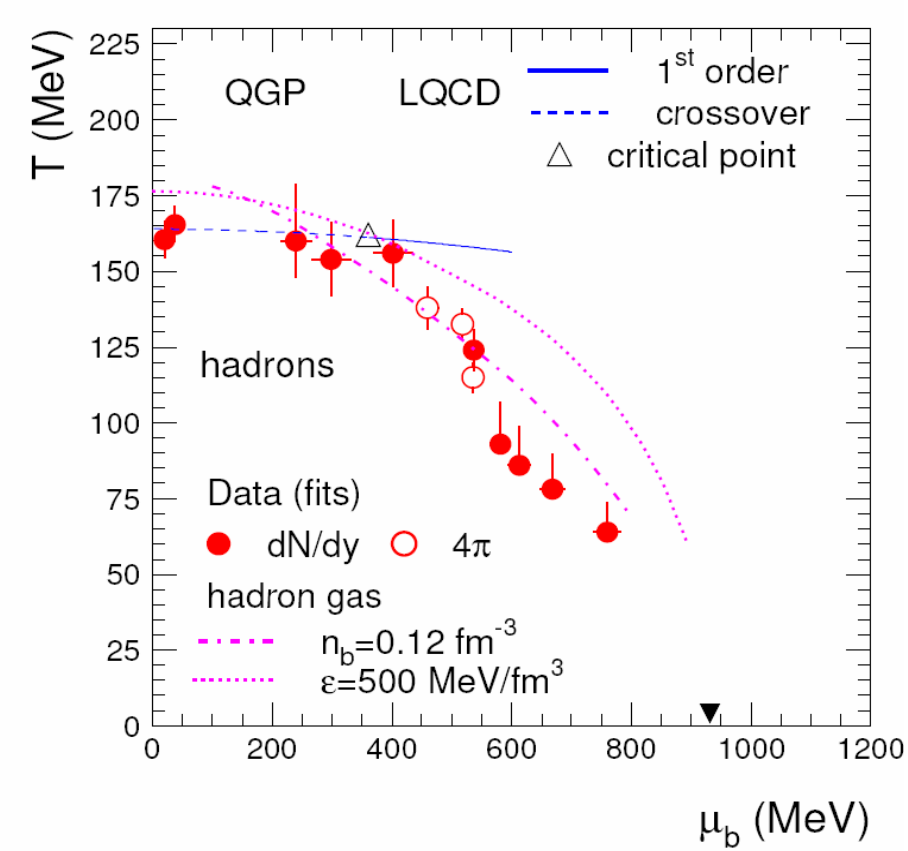
μ_B=0, T_c≈170 (10%) MeV



F.Karsch et al., Phys. Lett. B478 (2000) 447

T_{CP} =162(2) MeV, μ_{CP}=360(40) MeV

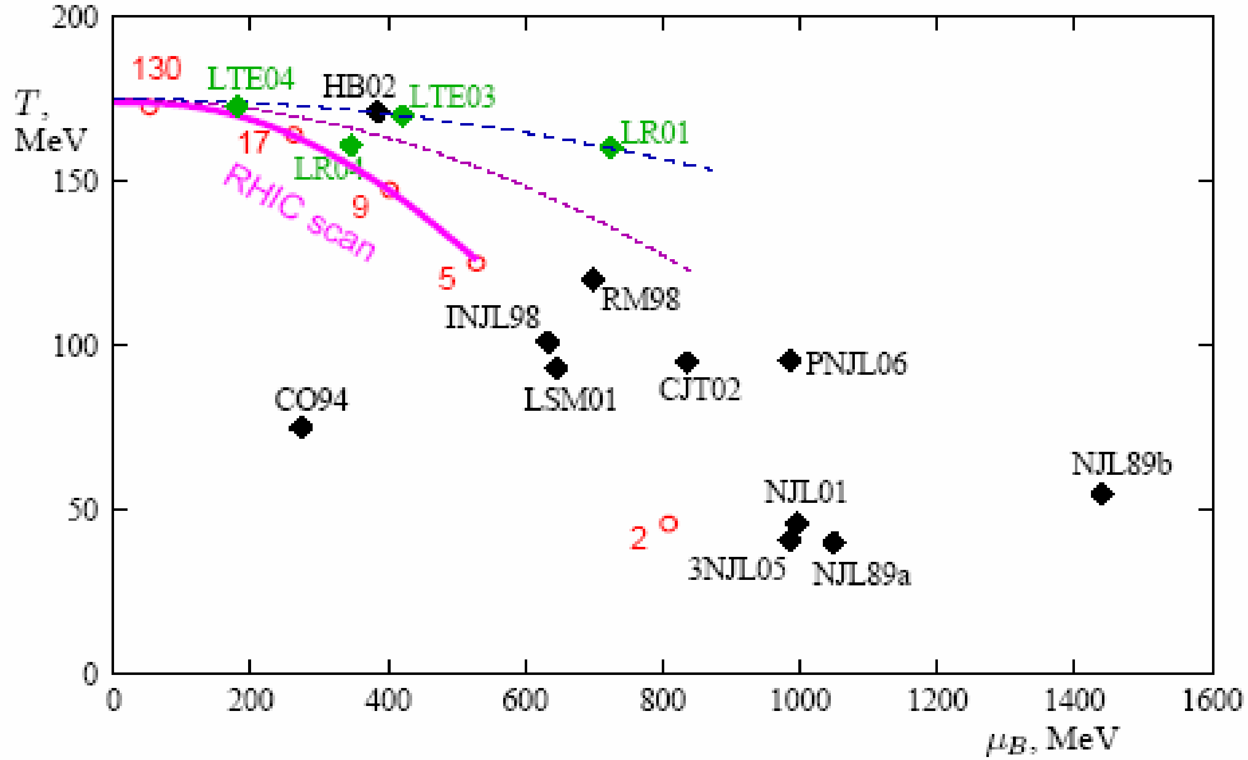
Z.Fodor,S.D.Katz, JHEP 404(2004)50



A.Andronic et al., Nucl.Phys. A772 (2006) 167

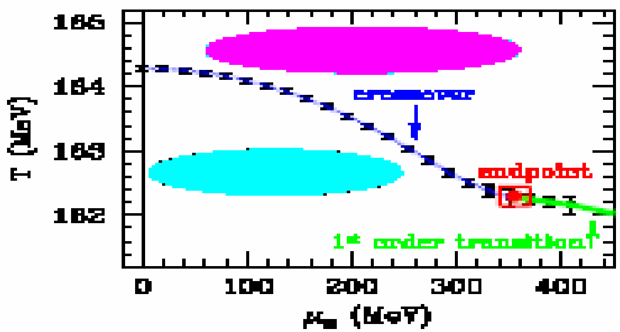
Locating the QCD critical points

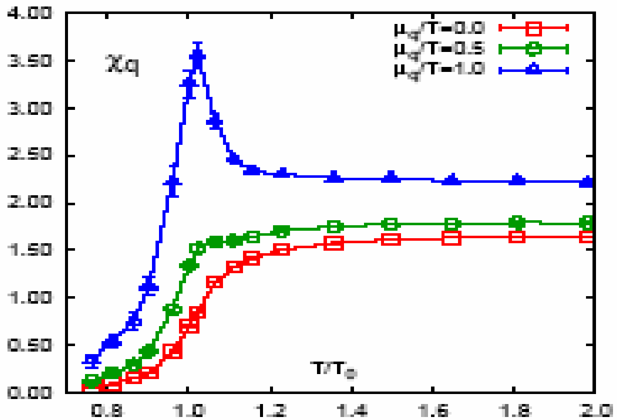
Models, lattice



Freezeout points from thermal model fits to SIS/AGS/SPS/RHIC data
(Braun-Munzinger, Redlich, Stachel)

Critical point on the lattice





There is a peak in χ_q but not in χ_I

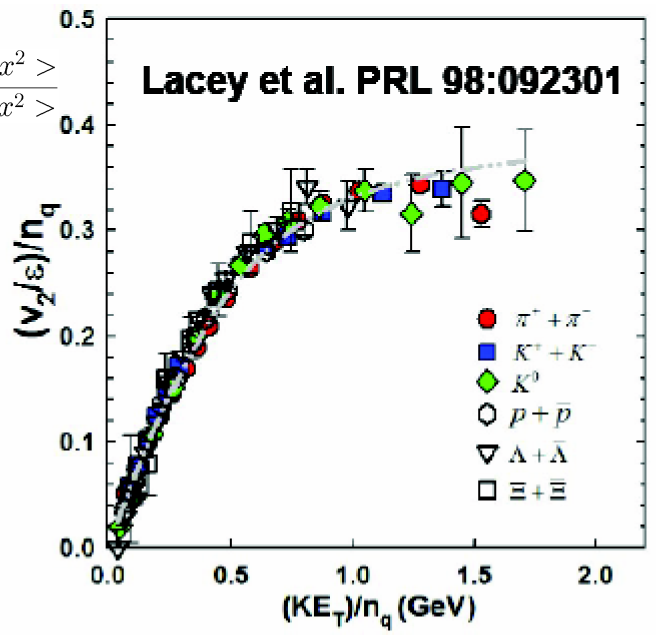
Several approaches:

- Reweighting: Fodor-Katz
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 360$ MeV (smaller m_q and larger V)
- Taylor expansion: Bielefeld-Swansea (to μ^6)
  - 2003: $\mu_B \sim 420$ MeV
  - 2005: 300 MeV $\lesssim \mu_B \lesssim$ 500 MeV
- Taylor expansion: Gavai-Gupta (to μ^8)
  - From convergence radius: $\mu_B \sim 180$ MeV (more precisely > 180 MeV)
- Imaginary μ: deForcrand-Philipsen, Lombardo, *et al*
  - Sensitive to m_s, perhaps $\mu_B \gg 300$ MeV
- Fixed density: deForcrand, Kratochvila; Density of states: Fodor, Katz, Schmidt.
  - ? ($N_f = 4$, small volumes)

Viscosity-to-entropy ratio

minimum bias Au+Au, √s=200 GeV

$$\varepsilon = \frac{<y^2> - <x^2>}{<y^2> + <x^2>}$$



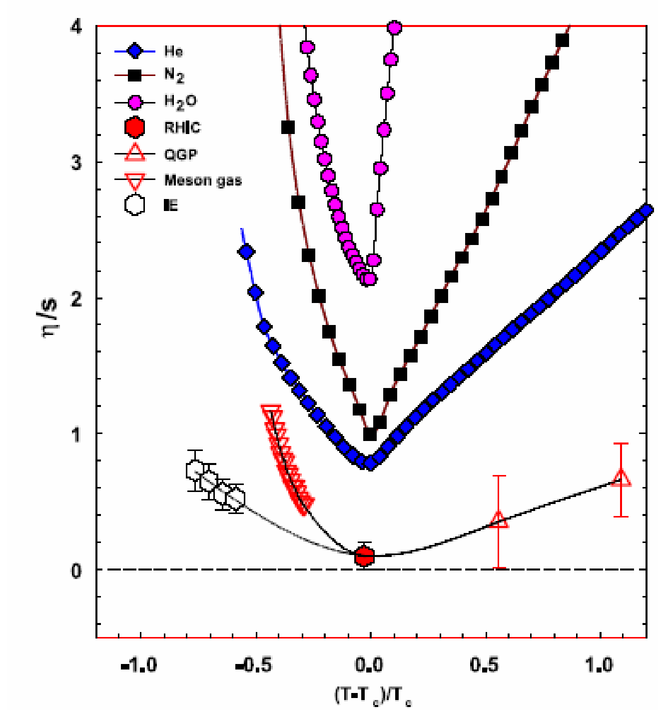
Hydrodynamic scaling

$$T^{ij} = \delta^{ij} P - \eta \left(\partial^i u^j + \partial^j u^i - \frac{2}{3} \delta^{ij} \partial_l u^l \right) - \zeta \delta^{ij} \partial_l u^l$$

Partonic fluid $\frac{\eta}{s} \sim T \lambda_f c_s,$ $\boxed{\sim 1.3 \times \left(\frac{1}{4\pi} \right)}$

Lower bound of η/s=1/4π in the strong coupling limit (P.Kovtun et al. PRL **94** (2005) 111601)

η/s for several substances

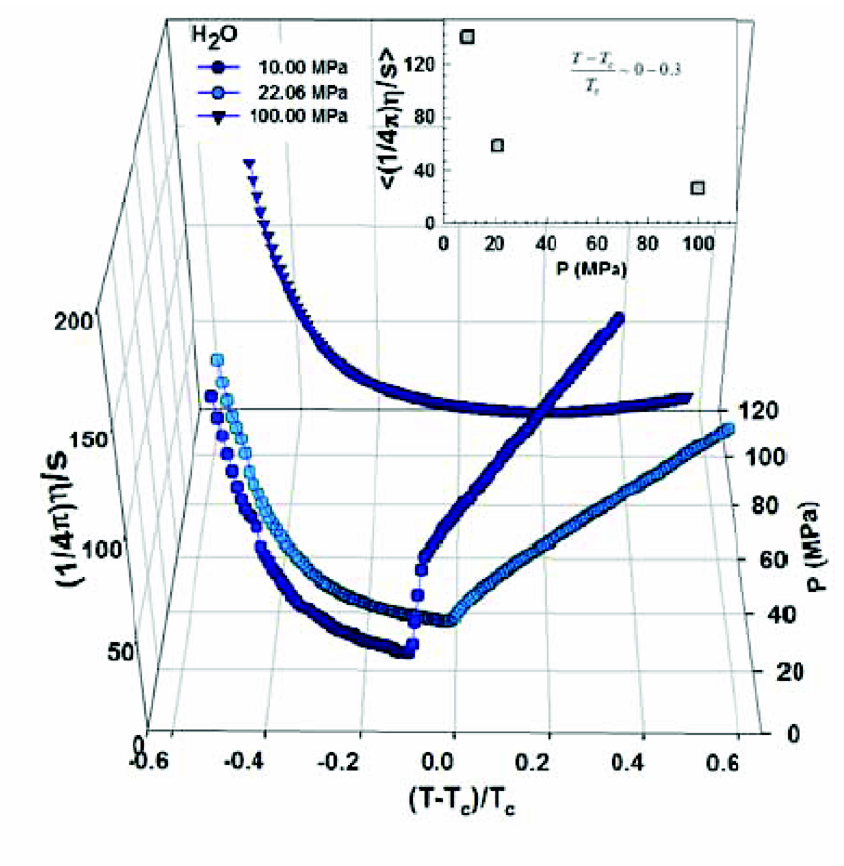


Strong indication for a minimum in the vicinity of T_c

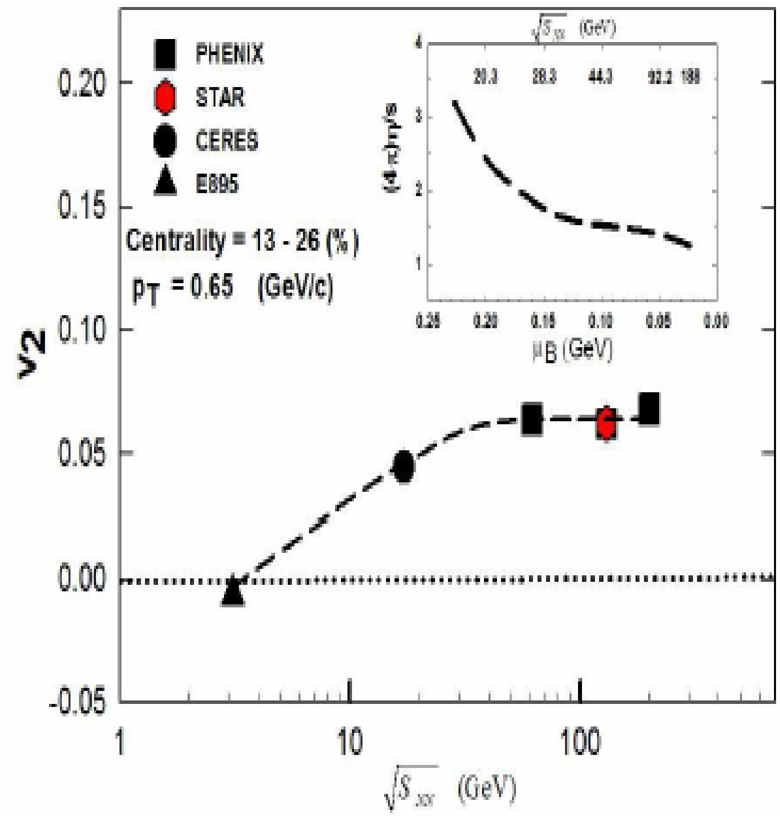
L.P.Csernai et al. PRL **97** (2006) 152303; R.Lacey at al. PRL **98** (2007) 092301

Location of the CEP (?)

Results for an isobar at the critical pressure P_c and one above/below it



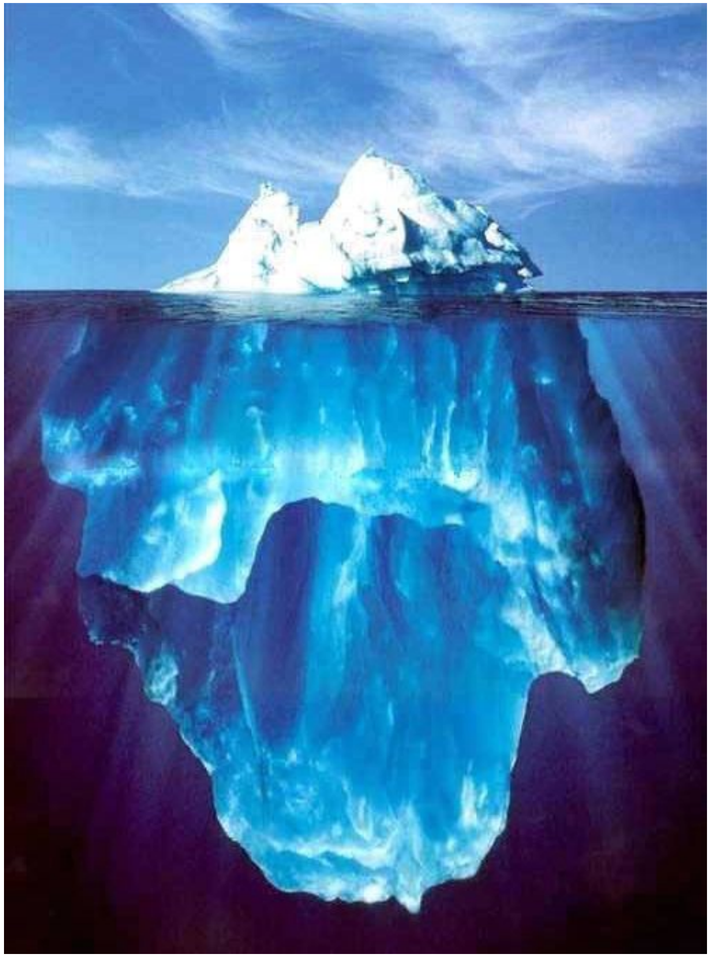
Flow excitation function



η/s is a potential signal of the CEP

R.Lacey et al. arXiv:0708.3512

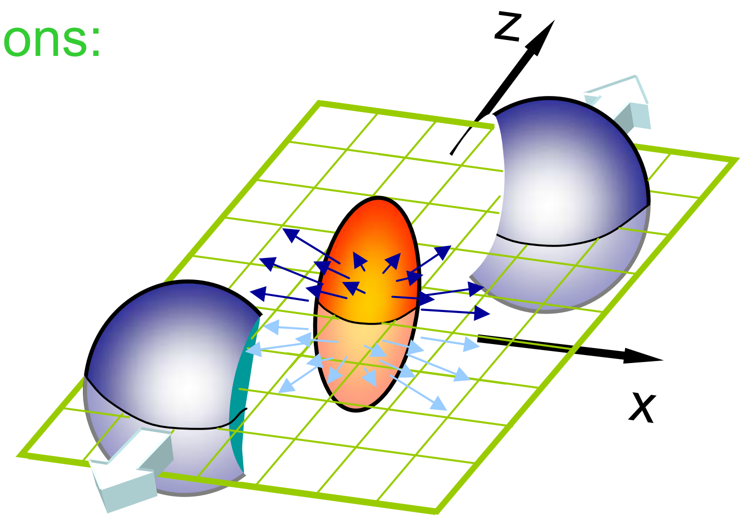
T-μ_B correlates at the freeze-out.
First esatimate T~175 MeV, μ_B~150 MeV



Thank you for attention !

Directed flow v_1 & elliptic flow v_2

Non-central Au+Au collisions:



Interactions between constituents leads to a pressure gradients => spartial asymmetry is converted in asymmetry in momentum space => collective flows

$$\frac{dN}{dy\,p_T\,dp_T\,d\varphi} = \frac{dN}{dy\,p_T\,dp_T}\frac{1}{2\pi}\left(1 + 2v_1\cos(\varphi) + 2v_2\cos(2\varphi) + \ldots\right)$$

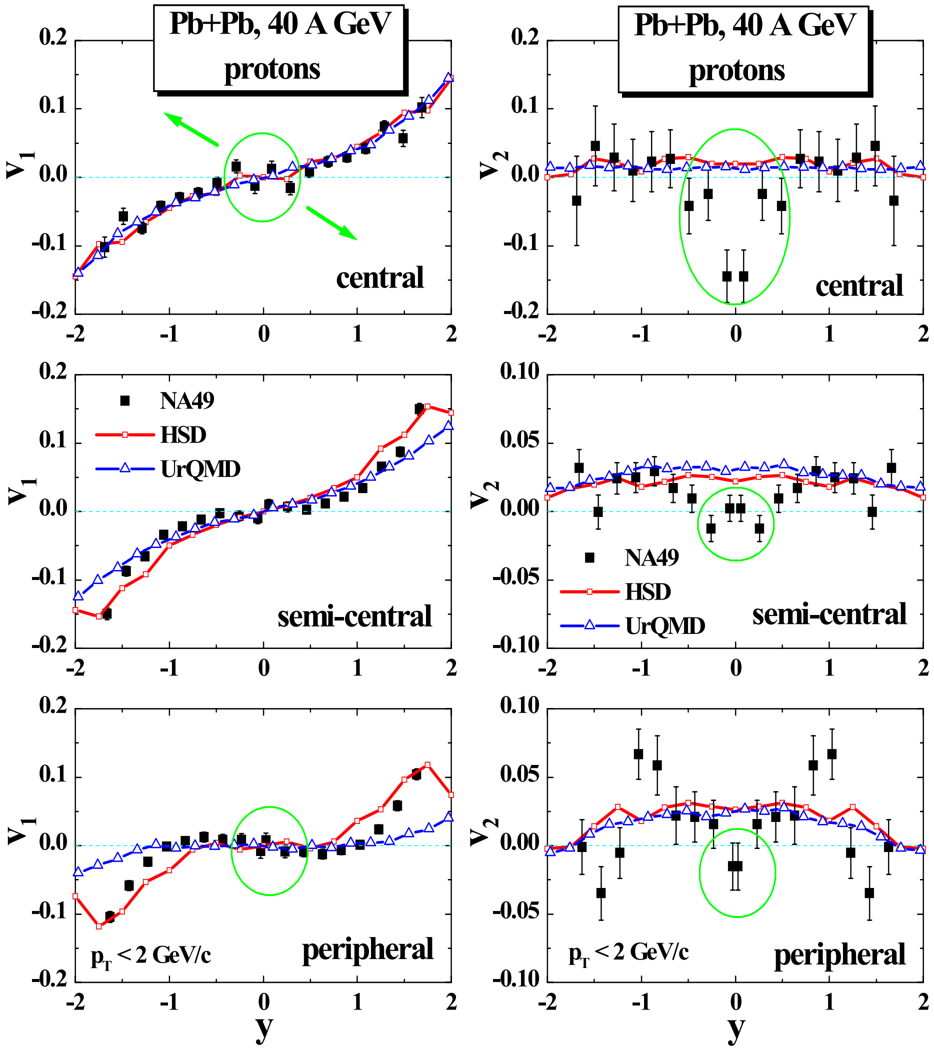
$$v_1 = <\frac{p_x}{p_T}>$$ - **directed flow**

$$v_2 = <\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}>$$ - **elliptic flow**

V_2>0 indicates in-plane emission of particles

V_2<0 corresponds to out-of-plane emission (squeeze-out perpendicular to the reaction plane)

v_1 and v_2 flows for Pb+Pb at 40 AGeV



Small wiggle in v_1 at midrapidity is not described by HSD and UrQMD

Too large elliptic flow v_2 at midrapidity from HSD and UrQMD for all centralities !

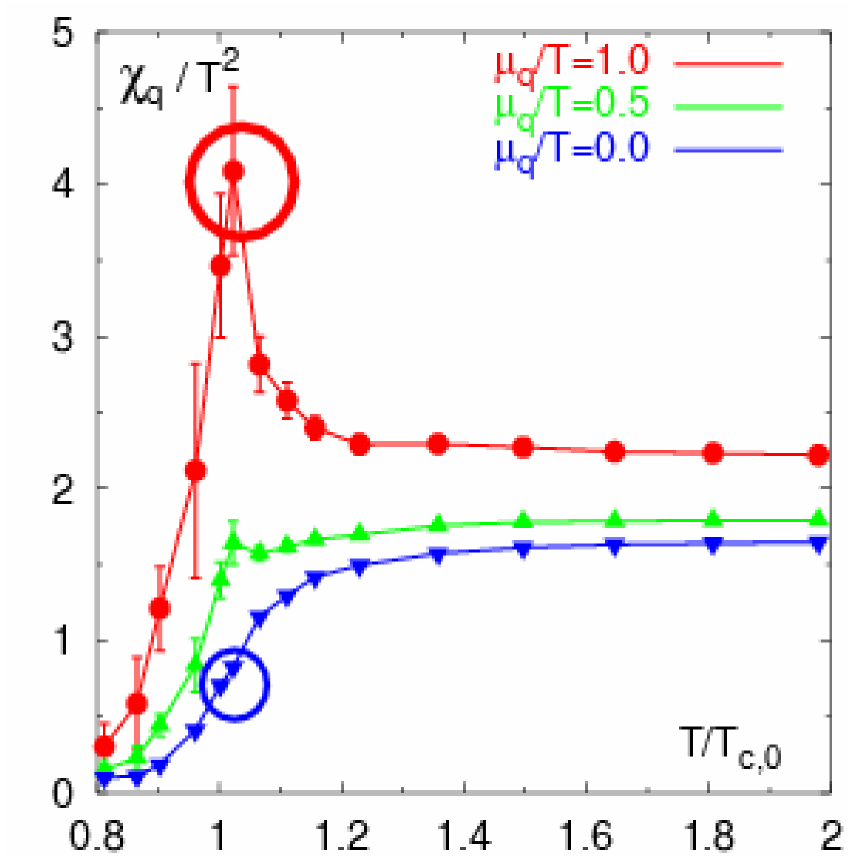
Experiment (NA49): breakdown of elliptic v_2 flow at midrapidity !

Signature for a first order phase transition

H.Stoecker et al., JPG 31 (2005) S929

Fluctuations

Lattice QCD predictions: Fluctuations of the quark number density (susceptibility) at $\mu_B > 0$ (C.Allton et al., PR D68 (2003) 014507)

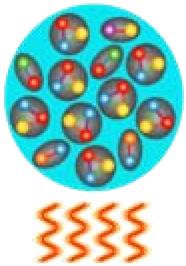


$$\frac{\chi_q}{T^2} = \left[\frac{\partial^2}{\partial(\mu_q / T)^2} \frac{P}{T^4} \right]_{T_{fixed}}$$

χ_q (quark number density fluctuations) will diverge at the critical end point

Experimental observation:

- Baryon number fluctuations
- Charge number fluctuations



heat

compression