Alpha-Clusters in Nuclear Systems

P. Schuck

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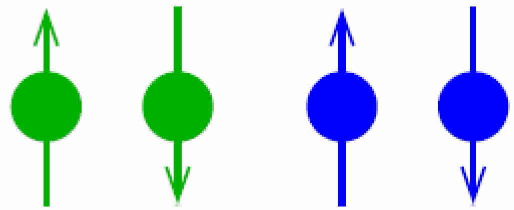
A. Tohsaki , W. von Oertzen and T. Yamada

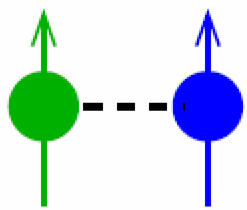
Contents:

- General Aspects of Nuclear Clustering
- α−Particles and their Condensation
- ^{8}Be and Hoyle State in ^{12}C*
- Extensions to Heavier Nuclei: ^{16}O
- α − de−excitation of Compound States
- α's in Compact Stars
- Conclusions, Outlook

Clusters important aspect and richness of nuclear systems due to 4 Fermions :

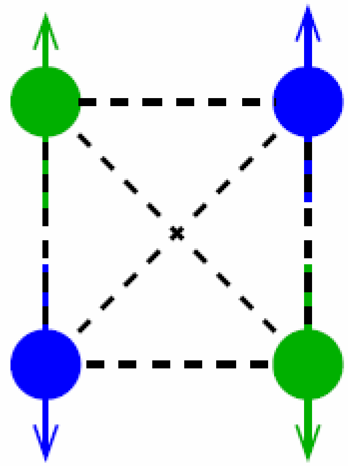


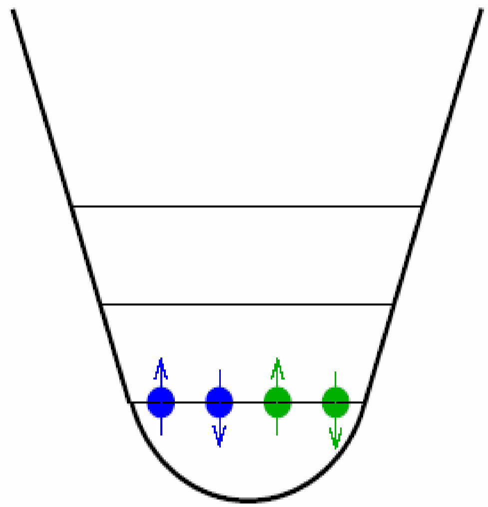
Dimer :

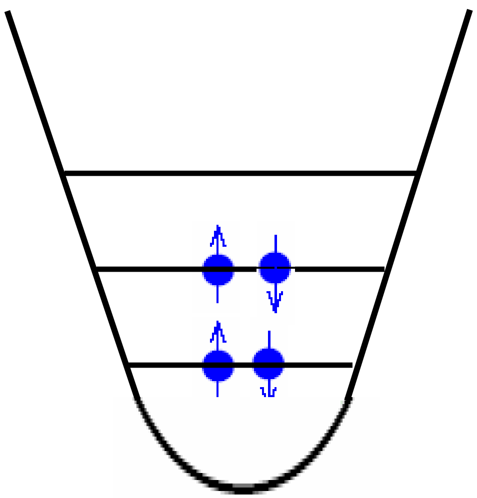


$$\frac{E}{A} = 1 \text{ MeV}$$

Quartet :





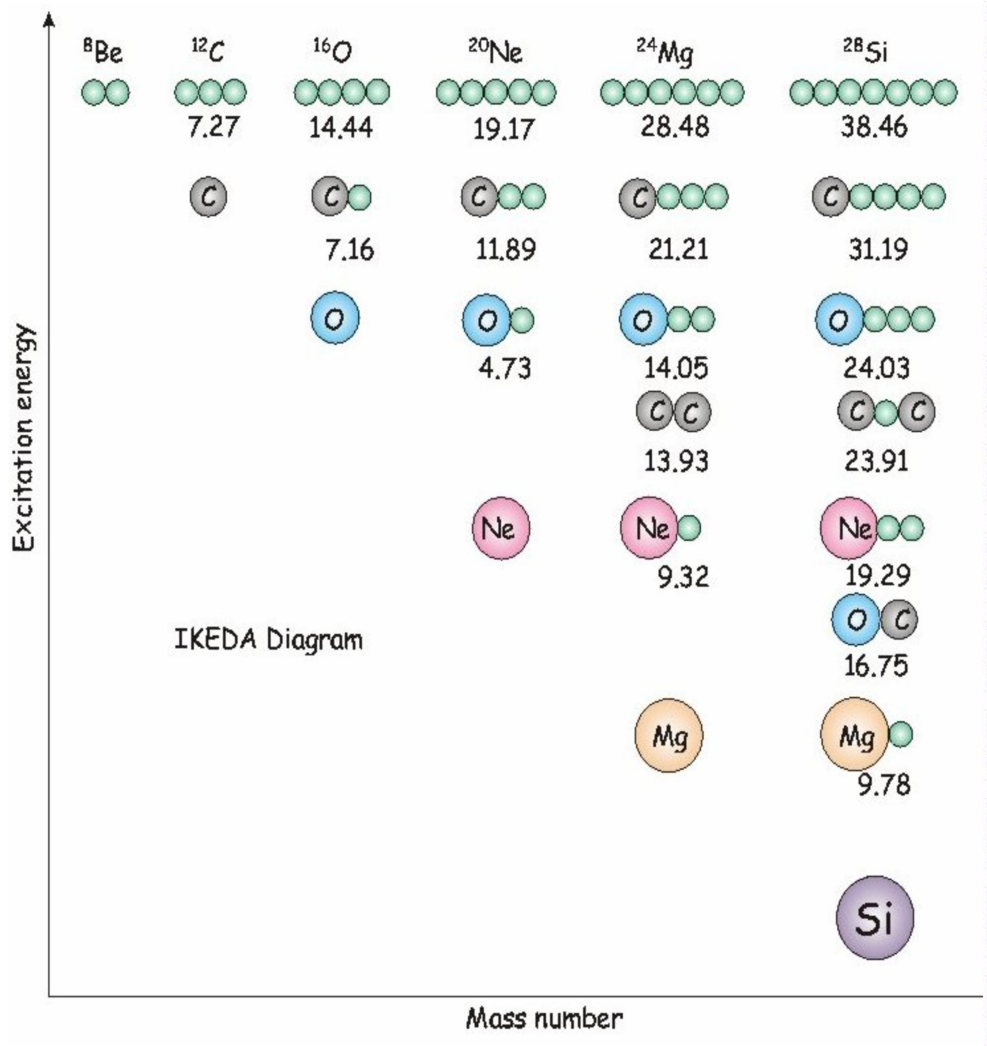


$$\frac{E}{A} = 7 \text{ MeV}, \quad E^{*} = 20 \text{ MeV}$$

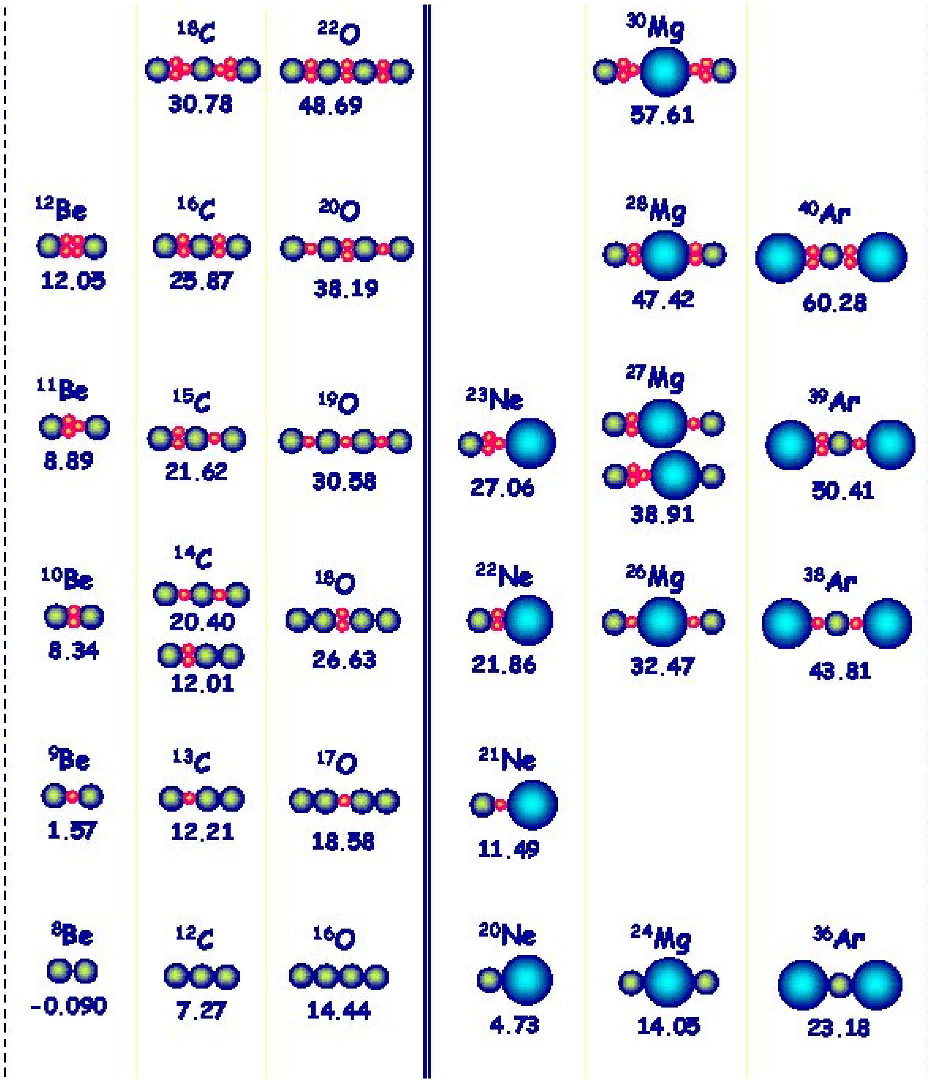
Proposal :

Trapping of 4 different species of Fermionic atoms.

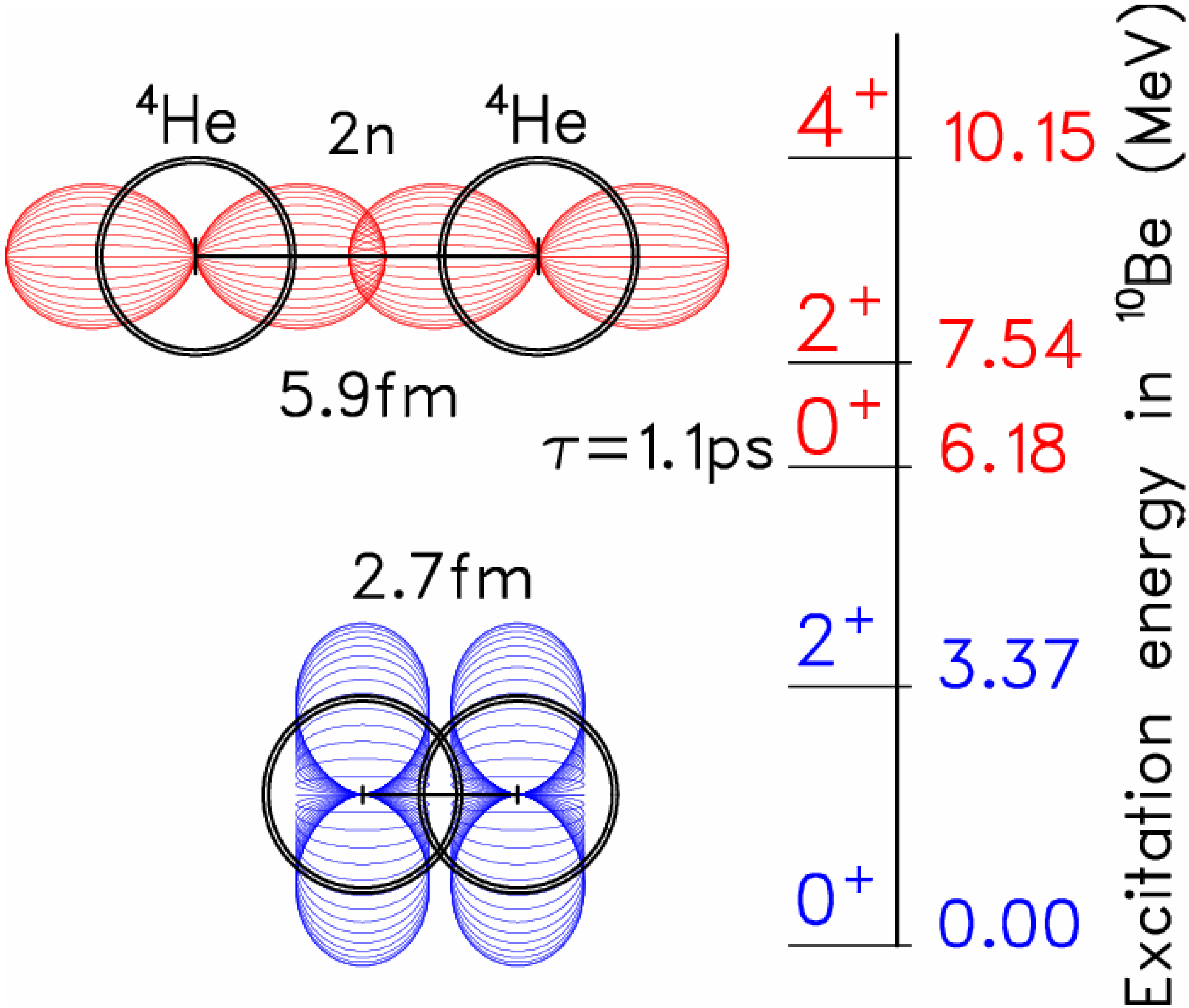
Ikeda



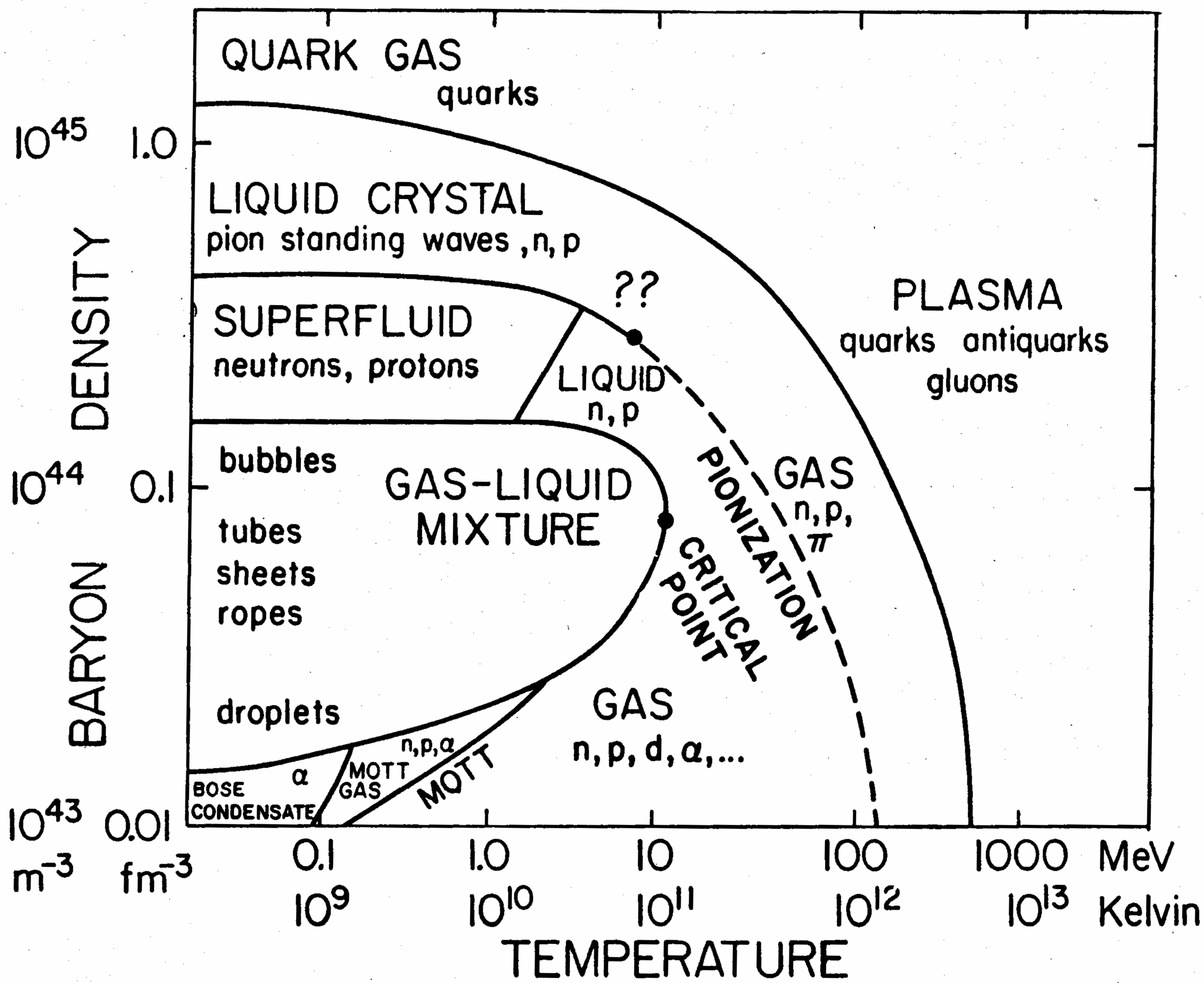
von Oertzen



H. G. Bohlen, M. Freer



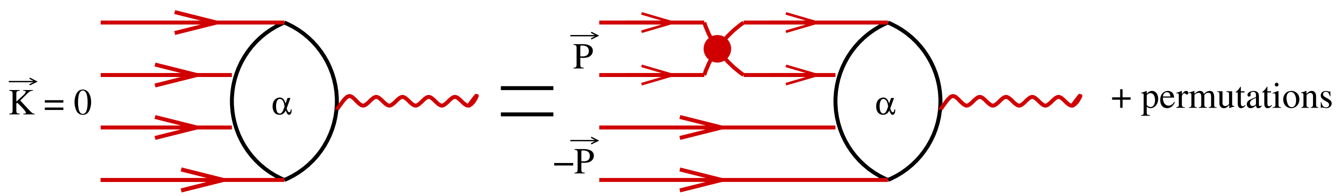
P. Siemens

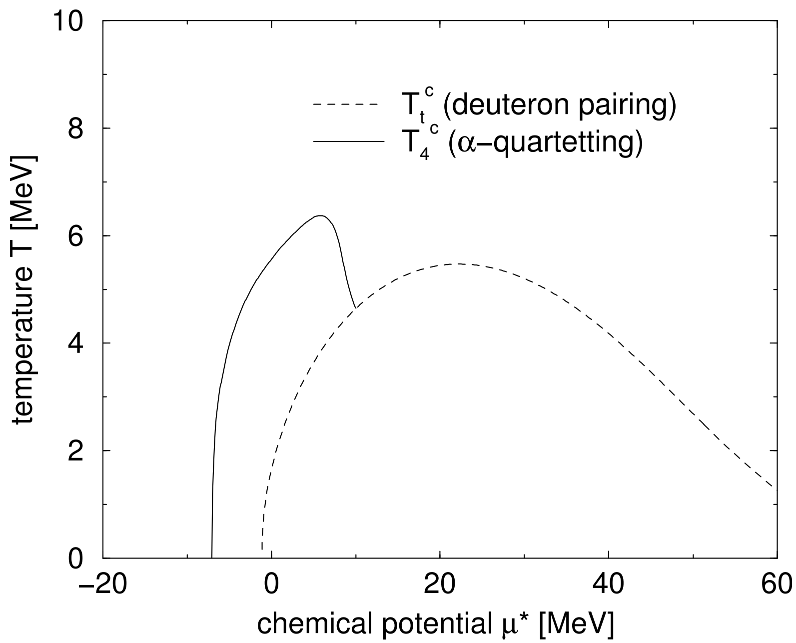


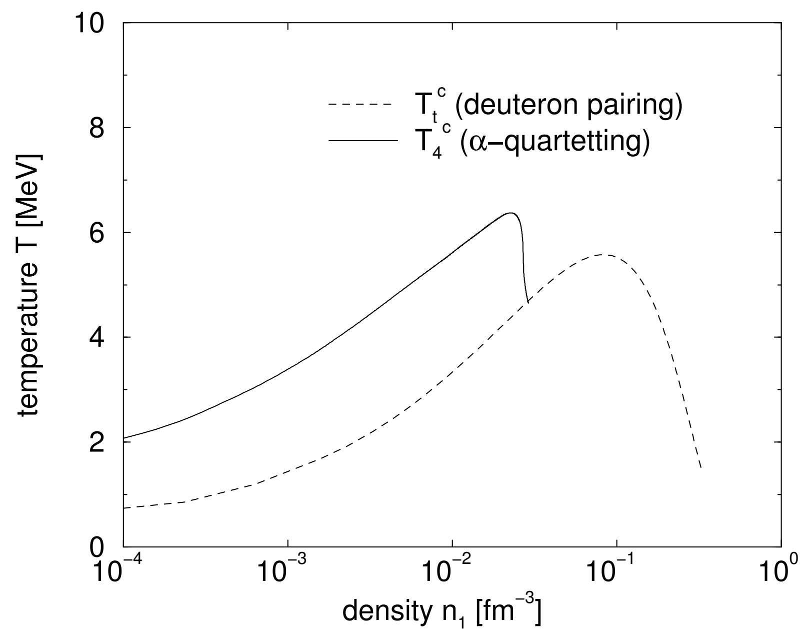
QUARK GAS
quarks
LIQUID CRYSTAL
pion standing waves ,n, p
SUPERFLUID
neutrons, protons
??
LIQUID
n, p
PLASMA
quarks antiquarks
gluons
bubbles
tubes
sheets
ropes
droplets
GAS-LIQUID
MIXTURE
CRITICAL
POINT
PIONIZATION
GAS
n, p, π
GAS
n, p, d, α,...
BOSE
CONDENSATE
α
MOTT
GAS
n,p,α
MOTT
BARYON DENSITY
10^45 1.0
10^44 0.1
10^43 0.01
m^-3 fm^-3
0.1 1.0 10 100 1000 MeV
10^9 10^10 10^11 10^12 10^13 Kelvin
TEMPERATURE

- α-Particle Condensation : G. Röpke, M. Beyer

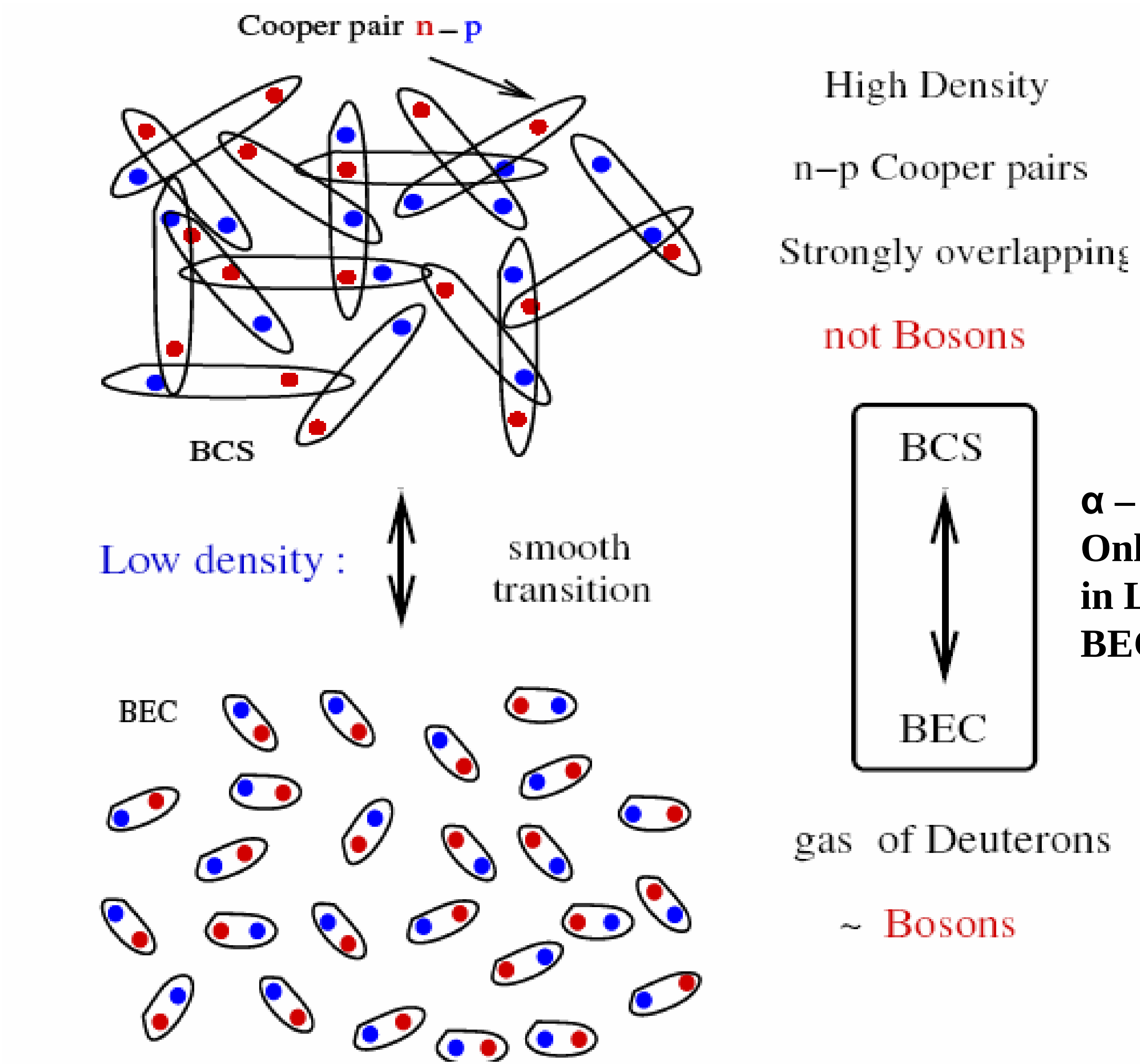
$$(4\mu - \epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4)\,\psi_{1234} = \quad (1 - f_1 - f_2)\sum_{1'2'} v_{121'2'}\psi_{1'2'34} + \text{permutations}$$







α-Condensation only at very low density !



α – Particles
Only Exist
in Low Density
BEC Phase

Finite nuclei ?

Exact ^{8}Be :

Density : $\frac{\rho_0}{3}$

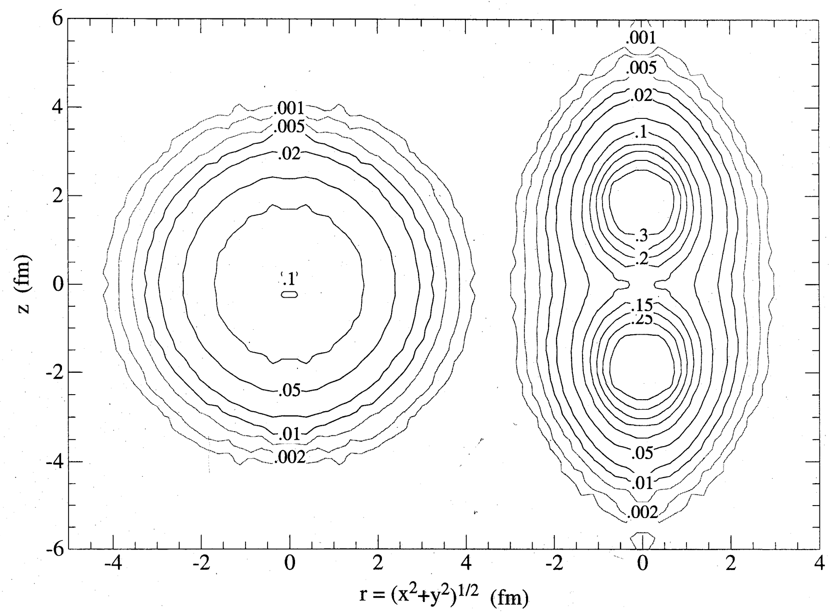
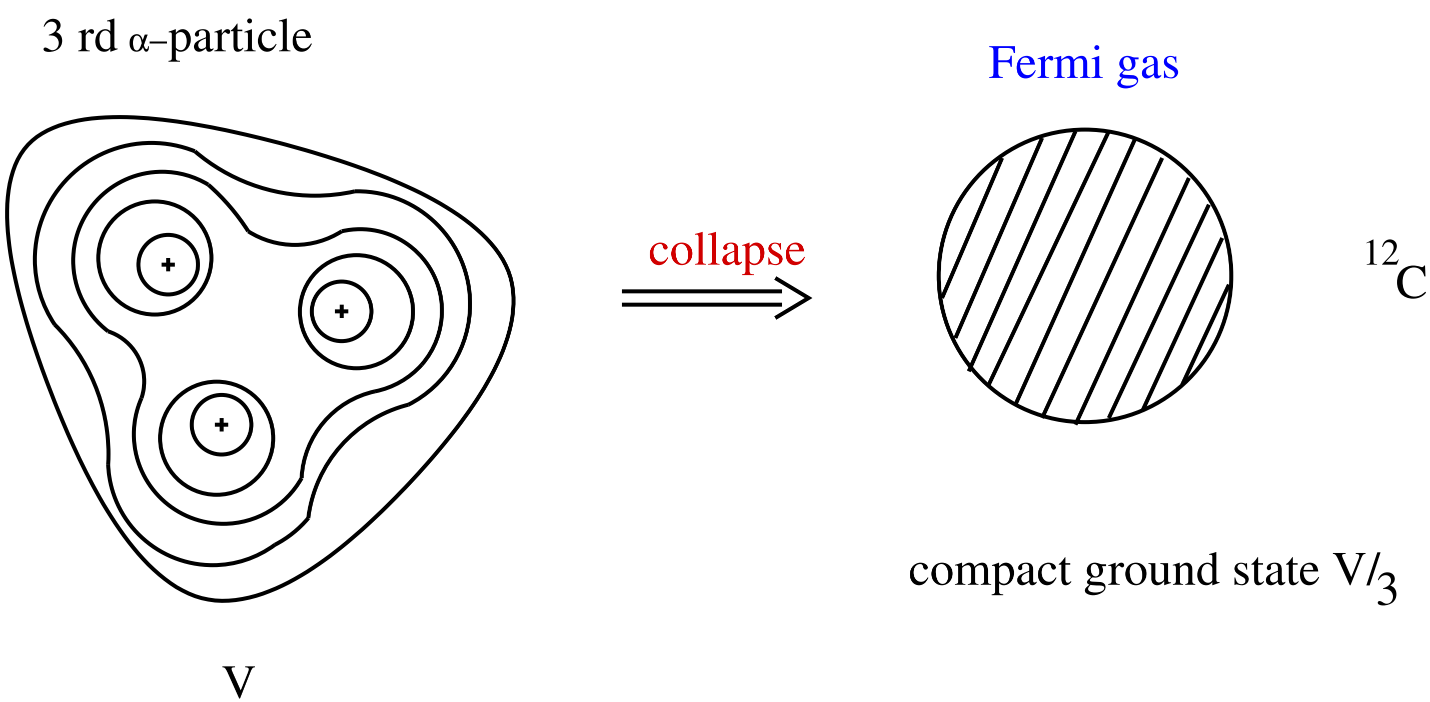
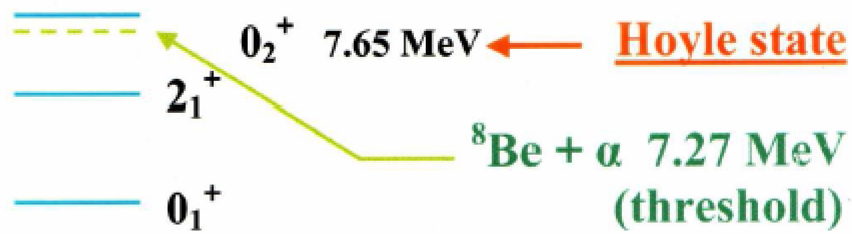


Fig. 15 (Wiringa, et al.)



Does a dilute 3α $^{12}C^*$ state exist ?
Similar to $^8Be + \alpha$?



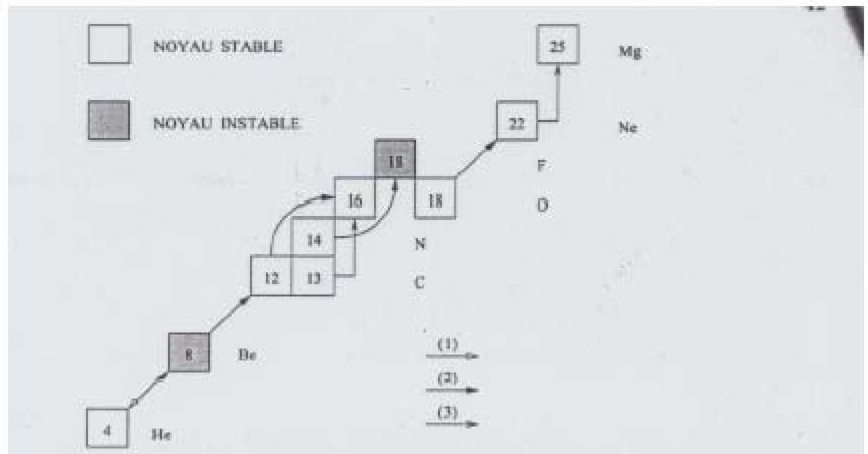


Figure 26. Réactions principales impliquées dans la combustion non explosive de He. Les flèches de type (1), (2) et (3) se rapportent respectivement à des réactions (γ, α), (α, γ), et (α, n)

At $T = 10^8 K$ helium burning
thermal equilibrium

$$\alpha + \alpha + \alpha \;\rightarrow\; {}^8Be \;\rightarrow\; \alpha \;\rightarrow\; {}^{12}C^* \quad O_2^+$$

O_2^+ : dilute 3α state hypothesis !

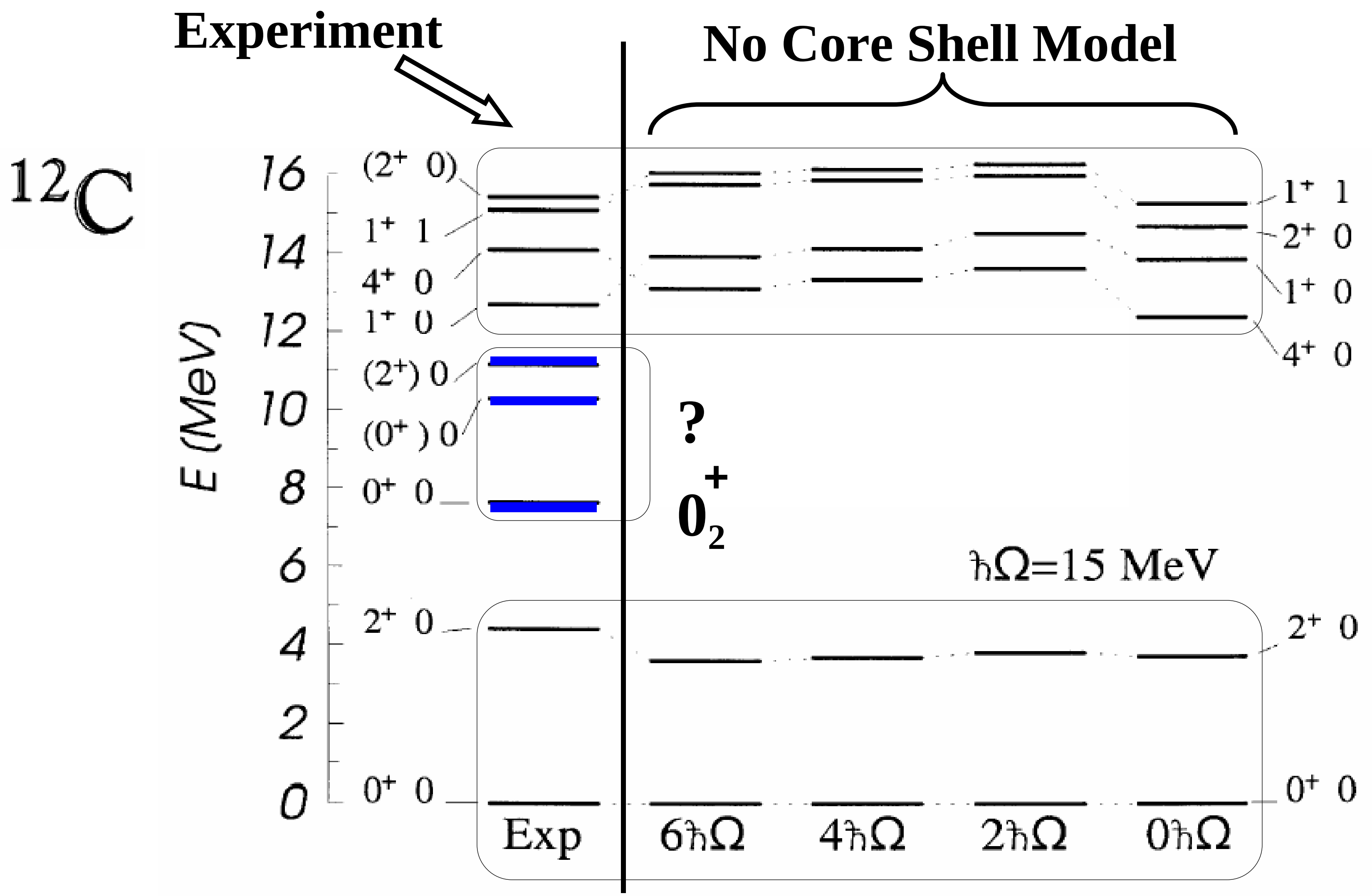
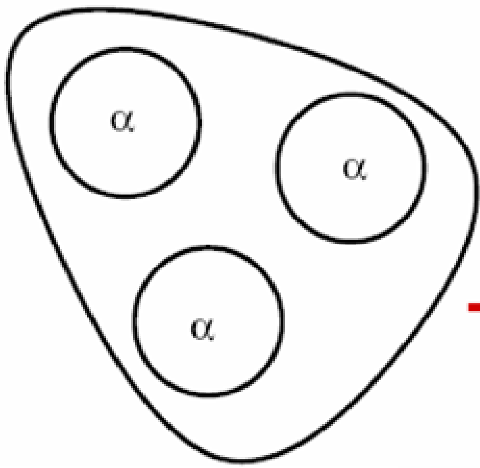


Figure 4. *Experimental and NCSM excitation spectra for ^{12}C for different model space sizes.*

it seems impossible to get Hoyle state from shell model calculation !

45 MeV B. Barret

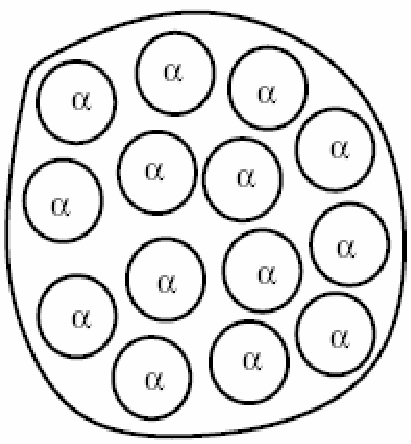
If O_2^+ in ^{12}C dilute $\alpha -$ state

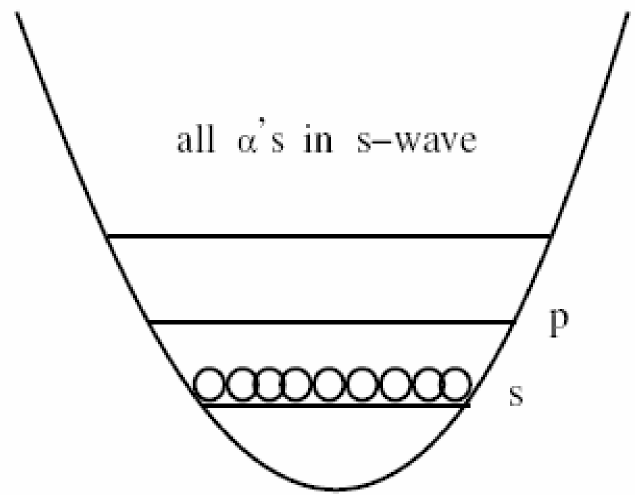


then α-condensate

infinite matter $\rho_{\text{crit}} \sim \frac{\rho_0}{3}$

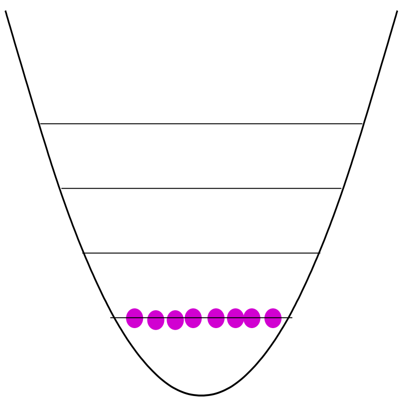
Conjecture: all n.α nuclei possess exited nα condensed state



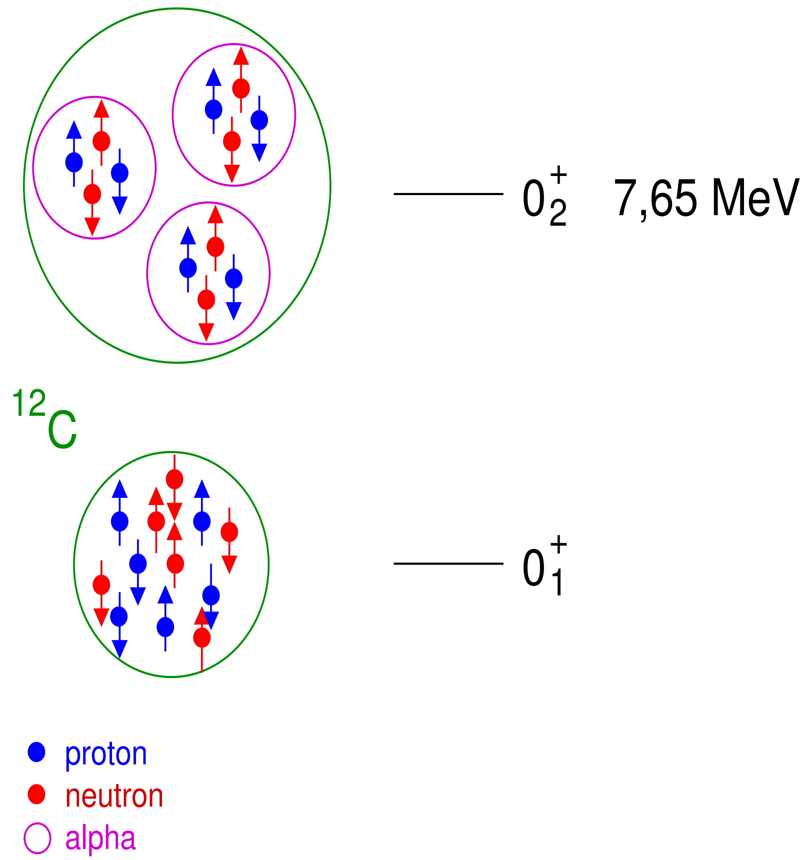


Analogy with atoms in traps ! $\rho(r) = N|\phi_0(r)|^2$ $N = 10^6$

Bosons

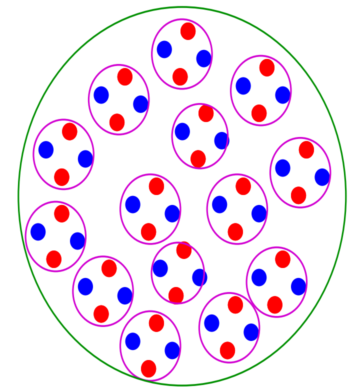


Back to nuclei

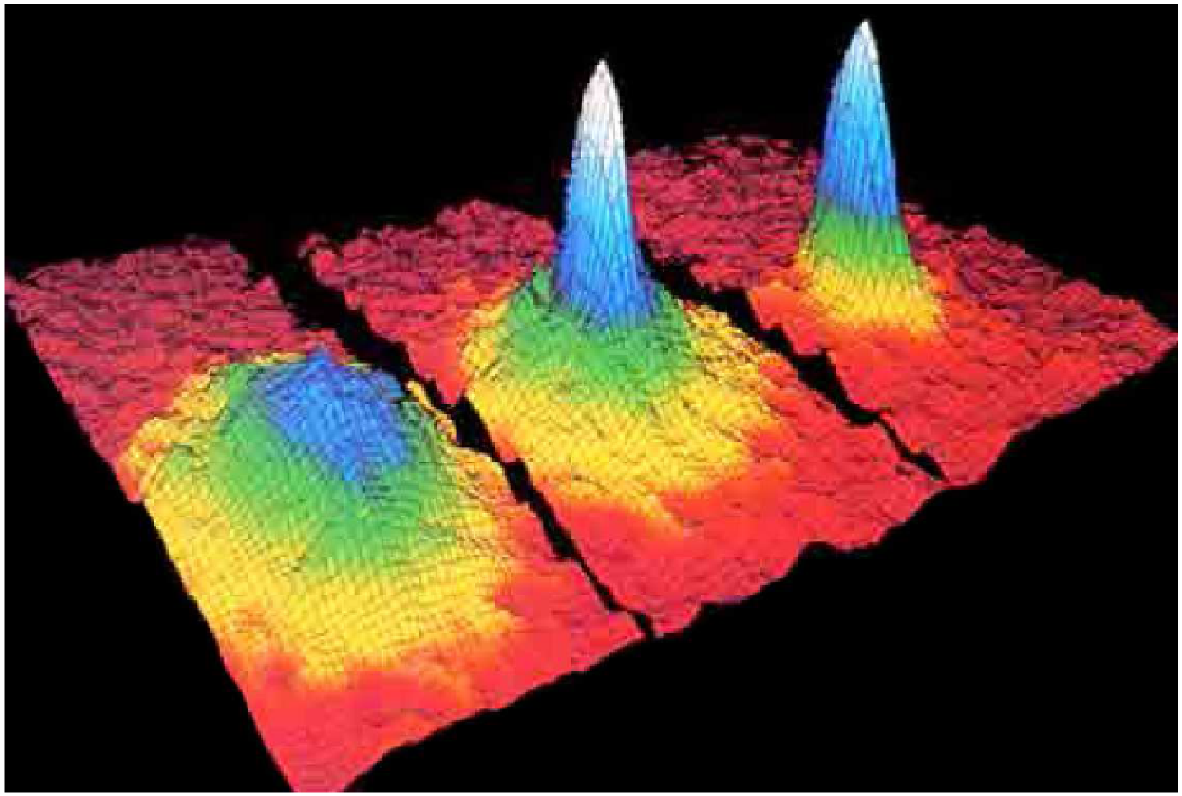


many α's

$\rightarrow$ condensate



strong cluster phenomena in lighter nuclei



Theoretical Description

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α-particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r-space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A}\left\{ \Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8) \cdots \Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n}) \right\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \mathrm{BCS} \rangle = \mathcal{A}\left\{ \Phi(\vec{r}_1, \vec{r}_2) \Phi(\vec{r}_3, \vec{r}_4) \cdots \right\}$$

Variational ansatz for $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$: $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = e^{-\frac{2}{B^2}\vec{R}^2}\phi_\alpha(\vec{r}_i - \vec{r}_j)$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$

Intrinsic α-wave function :

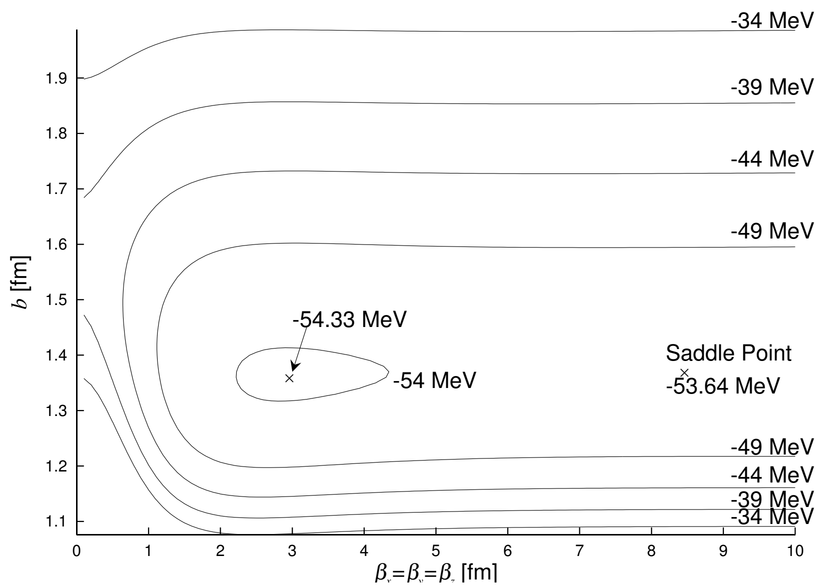
$$\phi_\alpha(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{8b^2}\left\{(\vec{r}_4-\vec{r}_1)^2+(\vec{r}_4-\vec{r}_2)^2+(\vec{r}_4-\vec{r}_3)^2+\cdots\right\}}$$

Two variational parameters : $B,\ b$

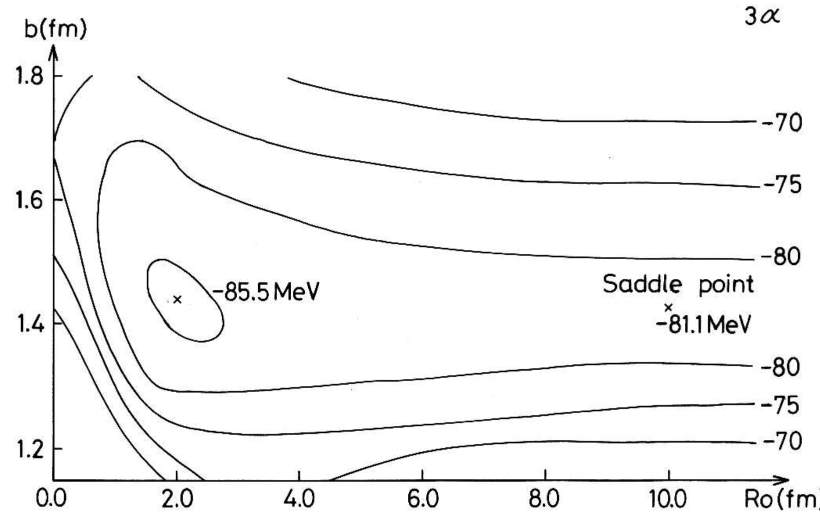
Two limits : $B = b$ $\quad |\Phi_{\alpha\mathrm{C}}\rangle =$ Slater determinant

$B \gg b$ $\quad |\Phi_{\alpha\mathrm{C}}\rangle =$ gas of independent α-particles

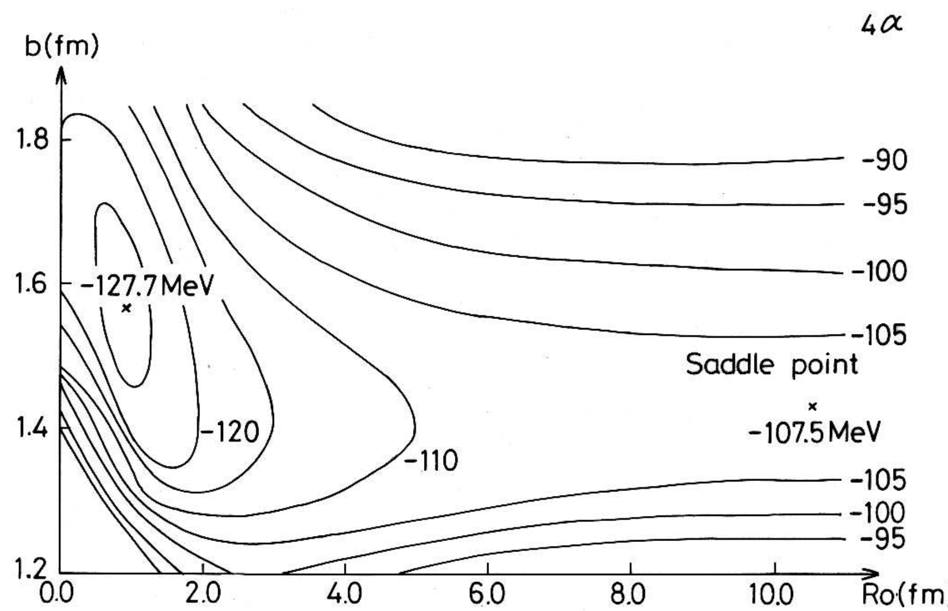
Two dimensional surface : $E(B,b) = \dfrac{\langle\Phi_{\alpha\mathrm{C}}|H|\Phi_{\alpha\mathrm{C}}\rangle}{\langle\Phi_{\alpha\mathrm{C}}|\Phi_{\alpha\mathrm{C}}\rangle}$


-34 MeV
-39 MeV
-44 MeV
-49 MeV
-54.33 MeV
-54 MeV
Saddle Point
-53.64 MeV
-49 MeV
-44 MeV
-39 MeV
-34 MeV
b [fm]
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0
1
2
3
4
5
6
7
8
9
10
$\beta_x=\beta_y=\beta_z$ [fm]

^{8}B


3α
b(fm)
1.8
1.6
1.4
1.2
-70
-75
-80
-85.5 MeV
Saddle point
-81.1 MeV
-80
-75
-70
0.0
2.0
4.0
6.0
8.0
10.0
Ro(fm)

^{12}C


4α
b(fm)
1.8
1.6
1.4
1.2
-90
-95
-100
-105
-127.7 MeV
-120
-110
Saddle point
-107.5 MeV
-105
-100
-95
0.0
2.0
4.0
6.0
8.0
10.0
Ro(fm)

^{16}O

Hamiltonian :

$$\begin{array}{ccccccccc} H & = & T & + & V_{N-N} & + & V_C & + & V_{N-N-N} \\ & & \text{Kin. energy} & & \text{Gaussien} & & \text{Coulomb} & & \text{Gaussian} \end{array}$$

Quantization of energy surface $E(B,b)$:

Force : A. Tohsaki ~ 1990 no adjustable parameters !

Hill-Wheeler : $|\psi\rangle = \sum_B f_B |\Phi_{\alpha C}(B)\rangle$

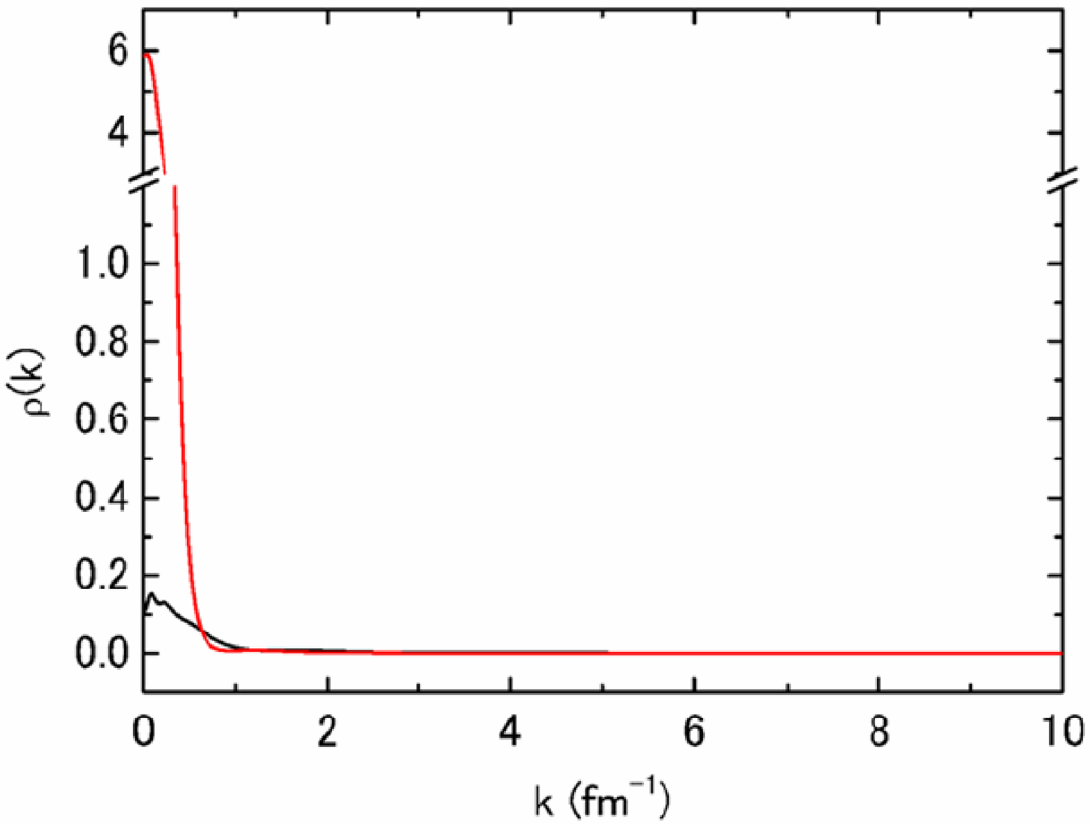
Without adjustable parameters :

$$\begin{array}{rcl} {}^{12}C: \quad (E_{O_2^+} - E_{3\alpha}) & = & \text{Theory} + 0.50 \text{ MeV} \\ & & \text{Exp.} + 0.38 \text{ MeV} \\ {}^{16}O: \quad (E_{O_5^+} - E_{4\alpha}) & = & \text{Theory} - 0.70 \text{ MeV} \\ & & \text{Exp.} - 0.44 \text{ MeV} \end{array}$$

r.m.s. of O_2^+ in ^{12}C 3.83 fm

ground state 2.40 fm

Momentum Distribution



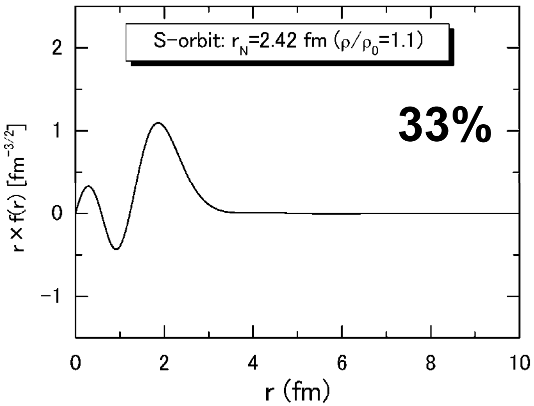
$$\frac{V_{O_2^+}}{V_{O_1^+}} \sim 3-4$$

	$Exp.$	$Cal.$
$M(O_2^+ \rightarrow O_1^+)$	$5.4 \pm 0.2 fm^2$	6.7
$R_{rms}(O_1^+)$	$2.43 fm$	2.40
$R_{rms}(O_2^+)$		3.47

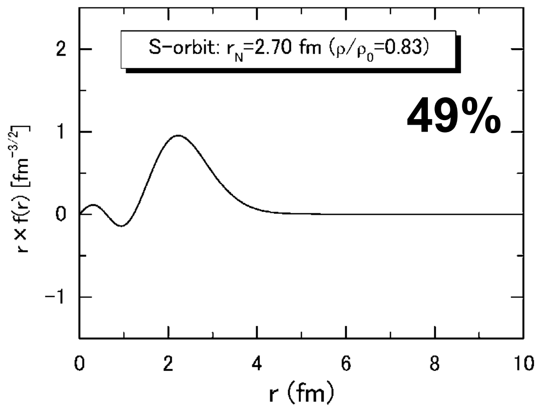
Radial behavior of S-wave α orbit vs. R_{rms}

R_{rms}=2.43 fm → 4.84 fm (ρ/ρ_0=1.1 → 0.14)

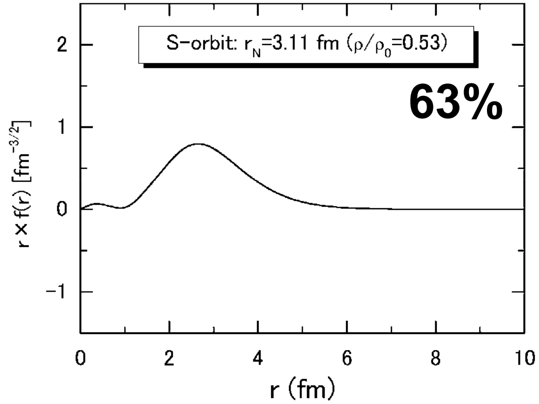
R_{rms}=2.43 fm (ρ/ρ_0=1.1)



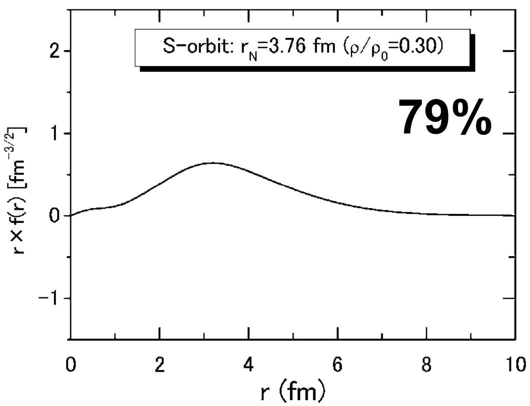
R_{rms}=2.70 fm (ρ/ρ_0=0.83)



R_{rms}=3.11 fm (ρ/ρ_0=0.53)



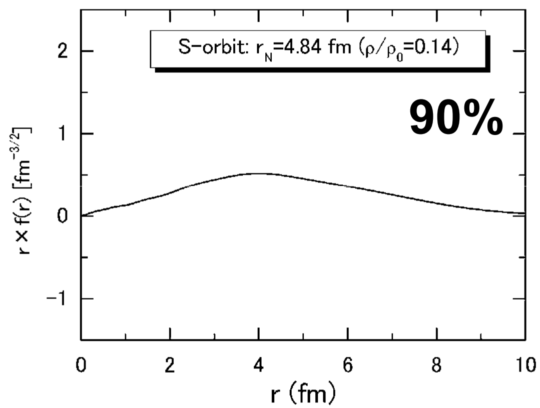
R_{rms}=3.76 fm (ρ/ρ_0=0.30)

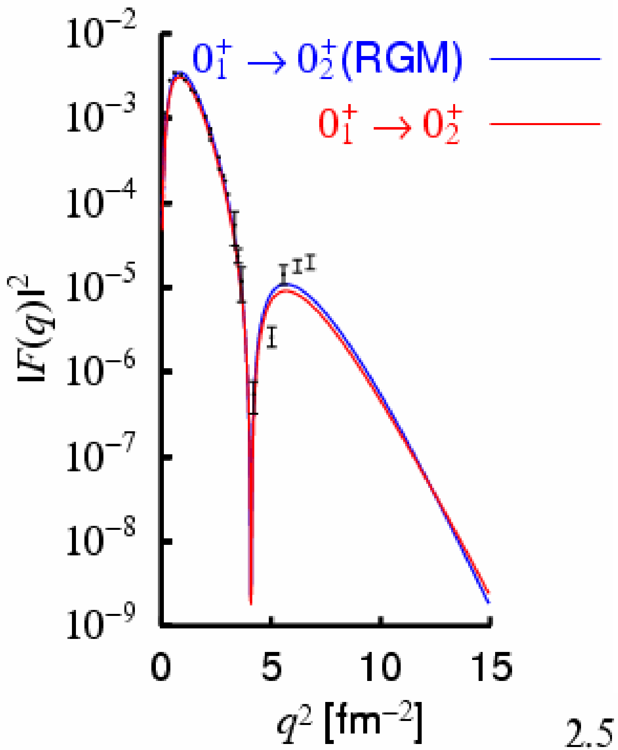


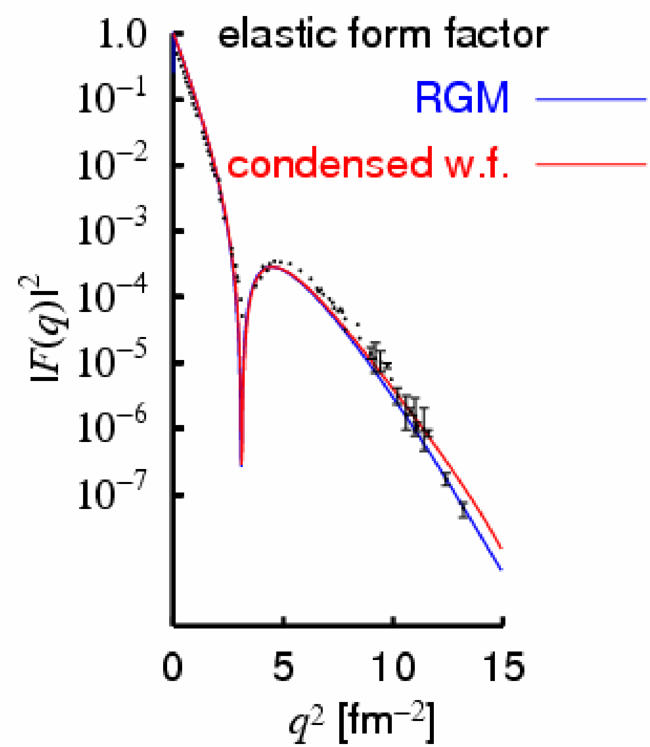
Increasing R_{rms},
we see smooth change
from 2S to 0S orbit.

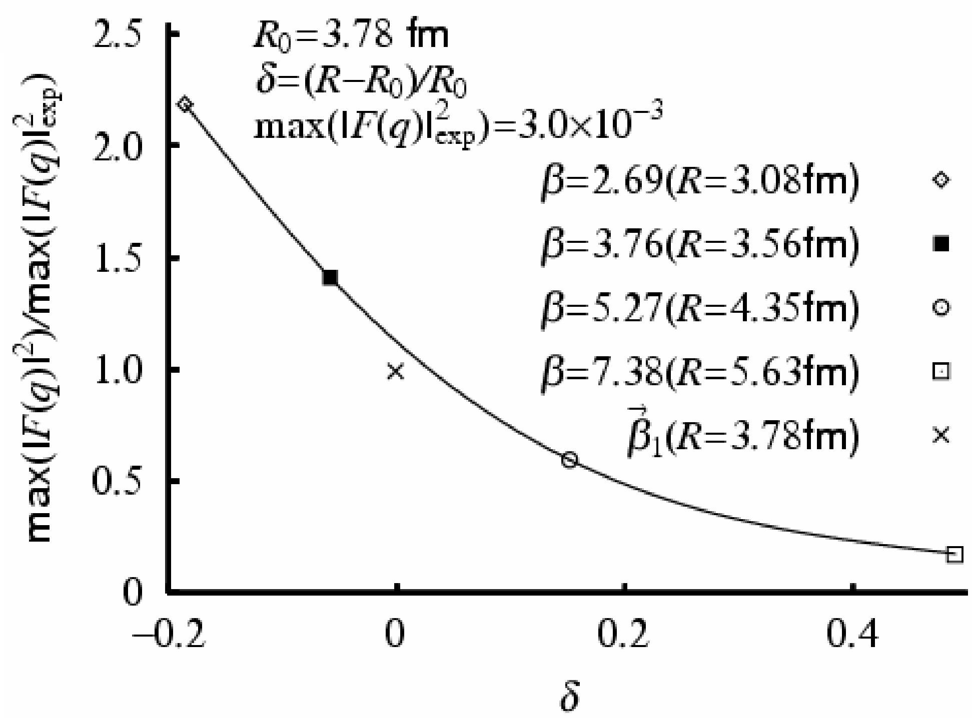
Yamada & Schuck, EPJA 2005

R_{rms}=4.84 fm (ρ/ρ_0=0.14)









R : r.m.s. radius

$$\vec{\beta}_1 := (\beta_x = \beta_y,\ \beta_z)$$
$$= (5.27\text{ fm}, 1.37\text{ fm})$$

Some more numbers :

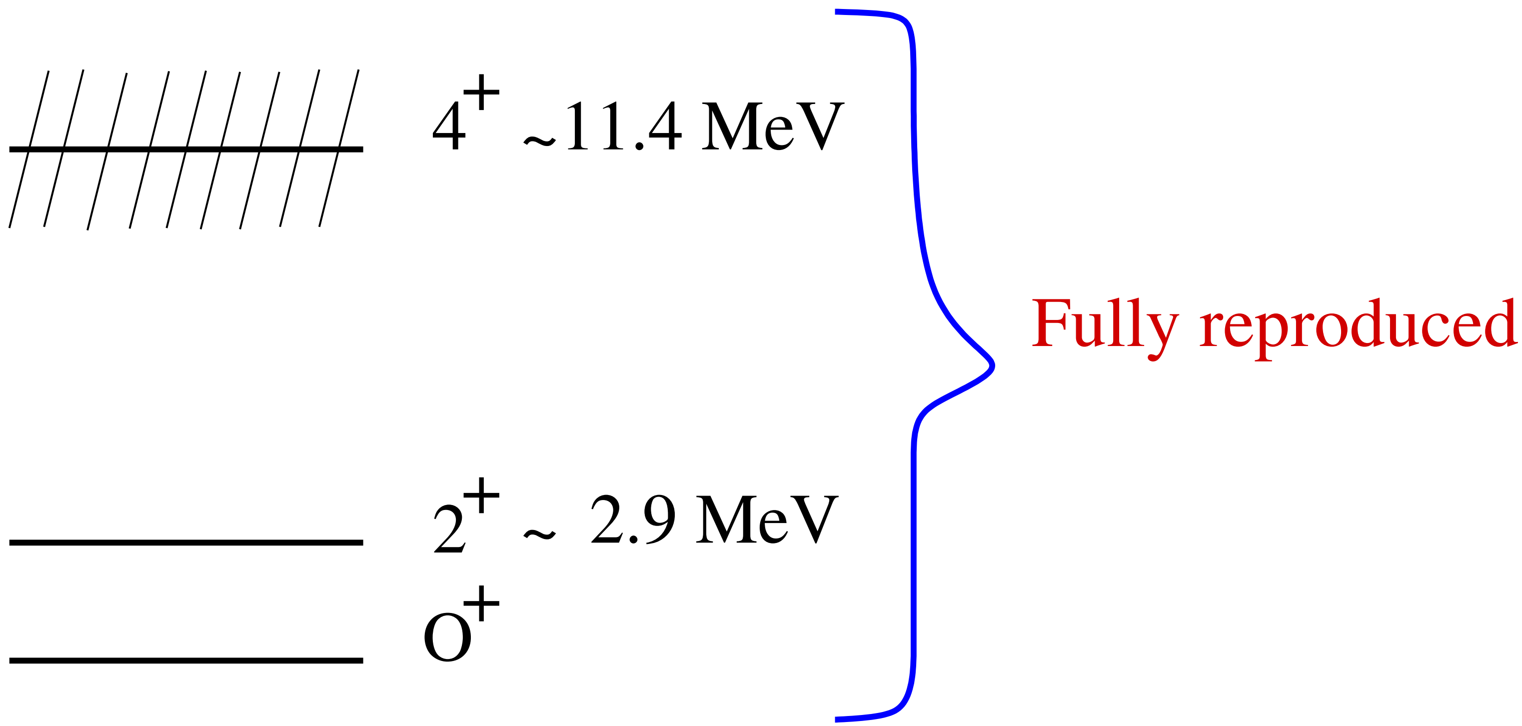
^{12}C :

	Theory	Exp.
O_1^+	-89.52	-92.16
O_2^+	-81.79	-84.51
	7.73	7.65

Threshold states $E - E_{\text{thresh.}}$

	^8Be	^{12}C	^{16}O	^{20}Ne
	2α	3α	4α	5α
theory	O_1^+	O_2^+	O_3^+	O_4^+
theory	-0.17	0.50	-0.7	1.8
experimental	O_1^+	O_2^+	O_5^+	?
experimental	0.09	0.38	-0.44	?

Spectrum of ^{8}Be :



^{12}C : Second excited 2^+ : 2_2^+

It has been discovered recently by Itoh *et al.*
2.6 MeV above 3 α rhreshold
Width ~ 1 MeV : resonance in continuum

Theory : We start with deformed α condensate state :

$$\Phi_{n\alpha} \propto \mathcal{A} \prod_{i=1}^{n} \exp\left\{ -\frac{2\,X_{ix}^2}{B_x^2} - \frac{2\,X_{iy}^2}{B_y^2} - \frac{2\,X_{iz}^2}{B_z^2} \right\} \Phi_{\alpha_i}$$

Then projection on good angular momentum

Then Hill Wheeler or GCM
For width : ACCC method

	Position	Width
Experiment :	2.6 ± 0.3 MeV	1.0 ± 0.3 MeV
Theory :	2.1 MeV	0.64 MeV

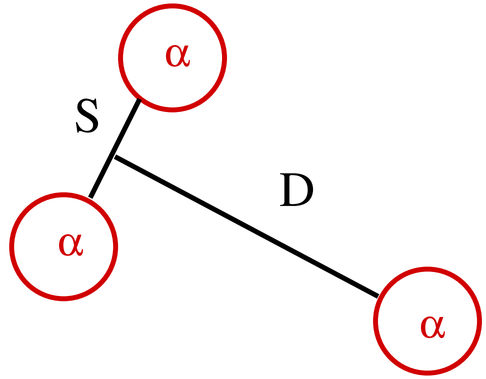
With in error bars !

RMS : $4.43\ fm$

$$\frac{V_{2_2^+}}{V_{O_1^+}} \sim 8 \quad !!$$

α – halo !

Internal structure :



Extremely dilute 3α state

Suggests a pure Boson picture $|\phi_0\rangle = b_0^+ b_0^+ b_0^+ \ldots |vac\rangle$

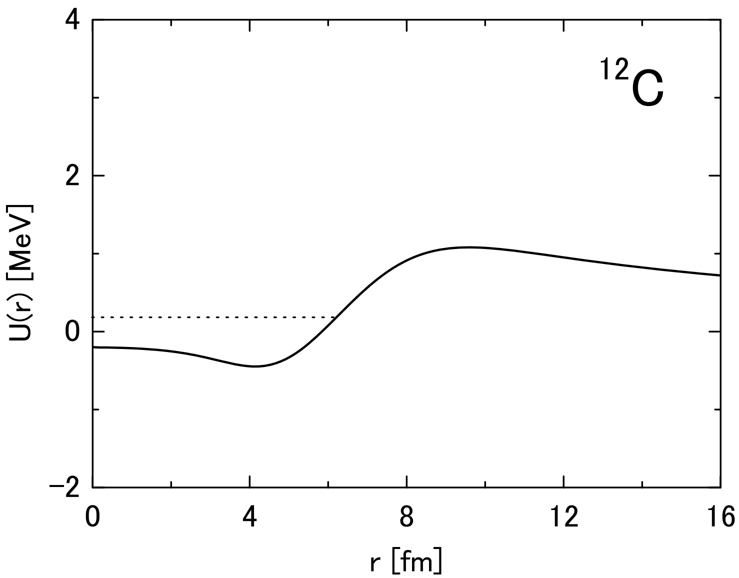
Hartree – Fock (Gross Pitaevsky eq)

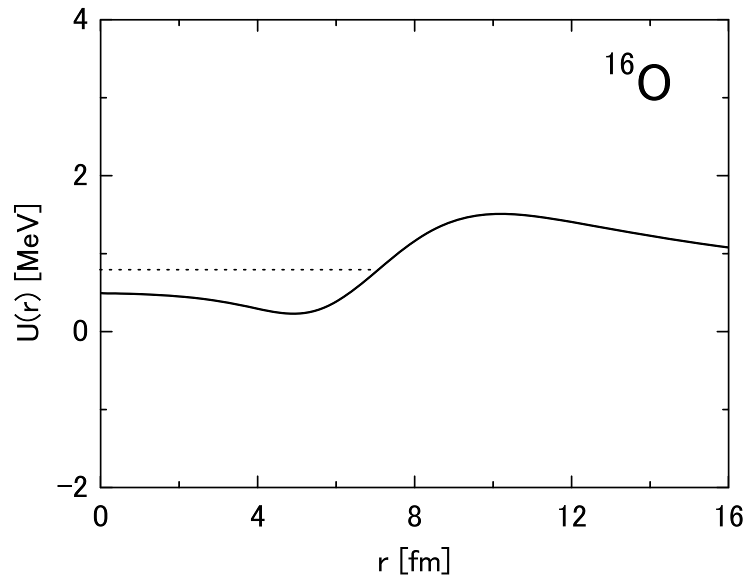
for ideal bosons (α' s) :

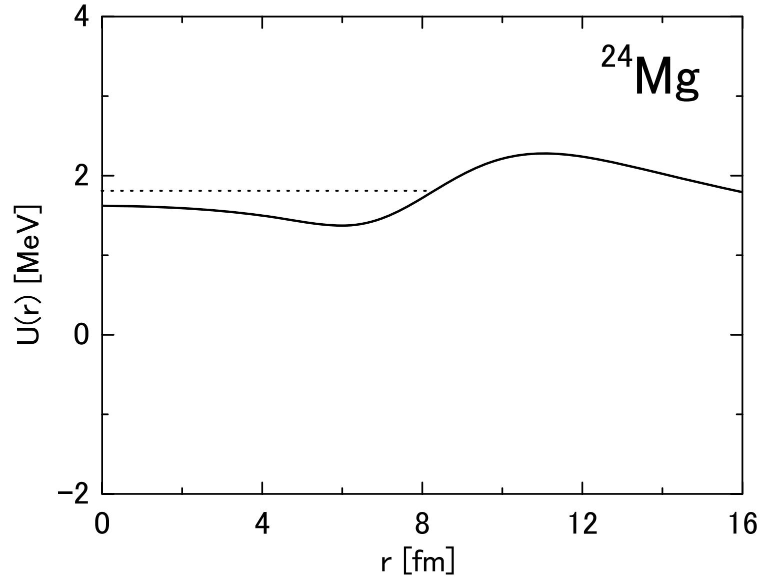
$$\left[-\frac{\hbar}{2m_\alpha}\Delta + N \int d^3r' v(\vec{r} - \vec{r'})|\phi_0(\vec{r'})|^2\right] \phi_0(\vec{r}) = \epsilon_0 \phi_0(\vec{r})$$

effective α – α + Coulomb

T. Yamada







Estimate for maximum number

$$\mathrm{N}^{\alpha}_{limit} \simeq 10 \qquad \Rightarrow \qquad {}^{40}\mathrm{Ca}^{**}$$

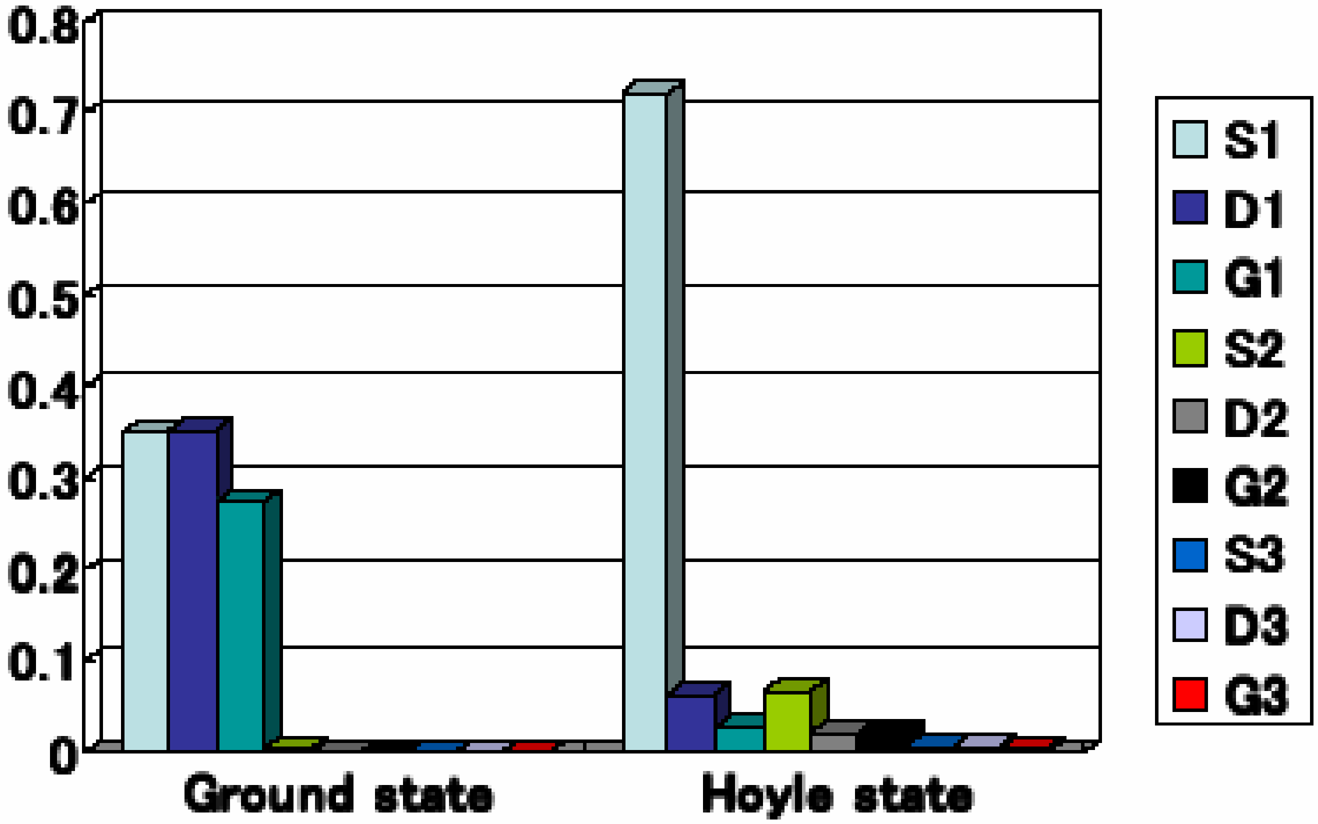
Boson occupancy :

α-particle density matrix :

$$\rho_\alpha(\vec{R}, \vec{R}'), \quad \vec{R} \; : \text{ c.m. of } \alpha$$

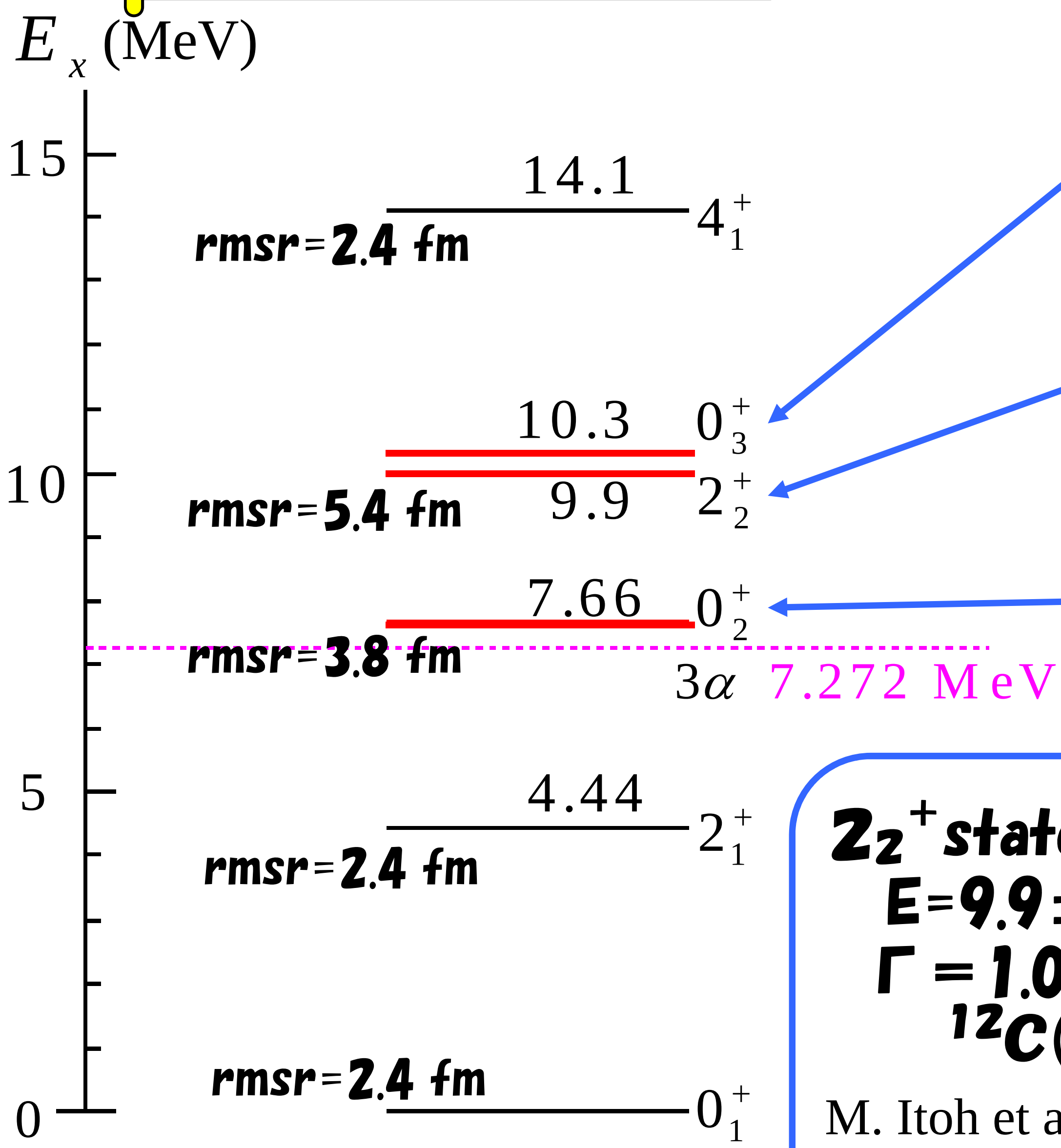
Diagonalization :

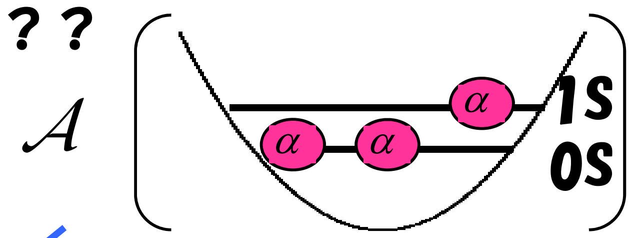
$$^{12}C : \quad O_2^+ \qquad 70\% \quad \text{S-wave occupancy}$$



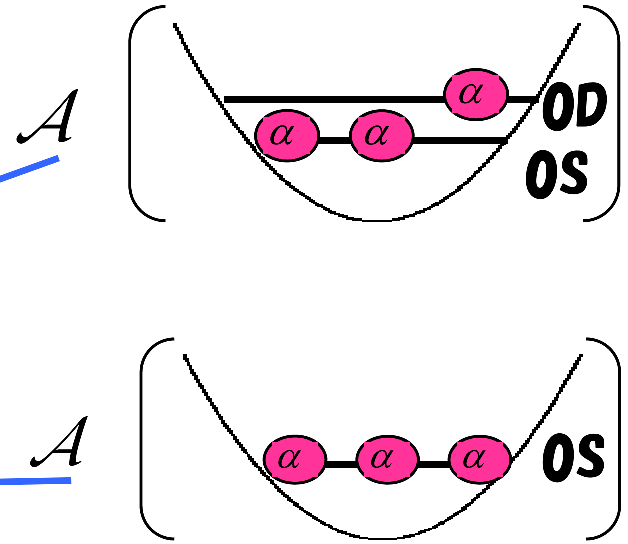
"BEC" in ^{12}C

Observed levels of ^{12}C





C. Kurokawa and K. Katō, PRC 71, 021301 (2005).



2_2^+ state :
E=9.9±0.3 MeV
Γ=1.0±0.3 MeV
$^{12}C(\alpha, \alpha')$

M. Itoh et al., Nucl. Phys. A 738 (2004) 268-272

α cond. +ACCC
E=9.38 MeV
Γ=0.64 MeV
Volkov No. 1 force is adopted

Y. F. et al., EPJA 24, 321 (2005).

Hamiltonian of 4α OCM

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

Pauli blocking operator on α - α motions

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{2n+\ell<4} \sum_{ij} \left|u_{n\ell}(r_{ij})\right\rangle\left\langle u_{n\ell}(r_{ij})\right|$$

Pauli forbidden state: h.o.w.f.

2-body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r}\,\mathrm{erf}\,(ar)$$

Phenomenological 3-body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i<j<k} \exp\left[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)\right]$$

$$V^{(3)} = 87.5 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Phenomenological 4-body force (repulsive)

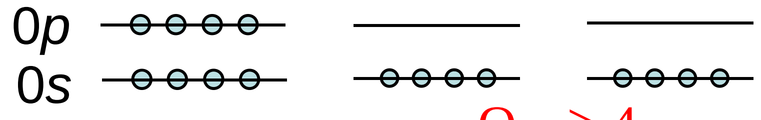
$$V_{4\alpha} = V^{(4)} \exp\left[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\right]$$

$$V^{(4)} = 12000 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Equation of motion

$$\delta\left[\left\langle \Phi_L(^{16}\mathrm{O})\right| H - E \left|\Phi_L(^{16}\mathrm{O})\right\rangle\right] = 0$$

$^{8}\mathrm{Be}(\mathrm{Q} \geq 4) \rightarrow \alpha(\mathrm{Q}=0) + \alpha(\mathrm{Q}=0)$



$\mathrm{Q}_{\mathrm{rel}} \geq 4$

α - α motion with $Q_{rel} < 4$ should be eliminated

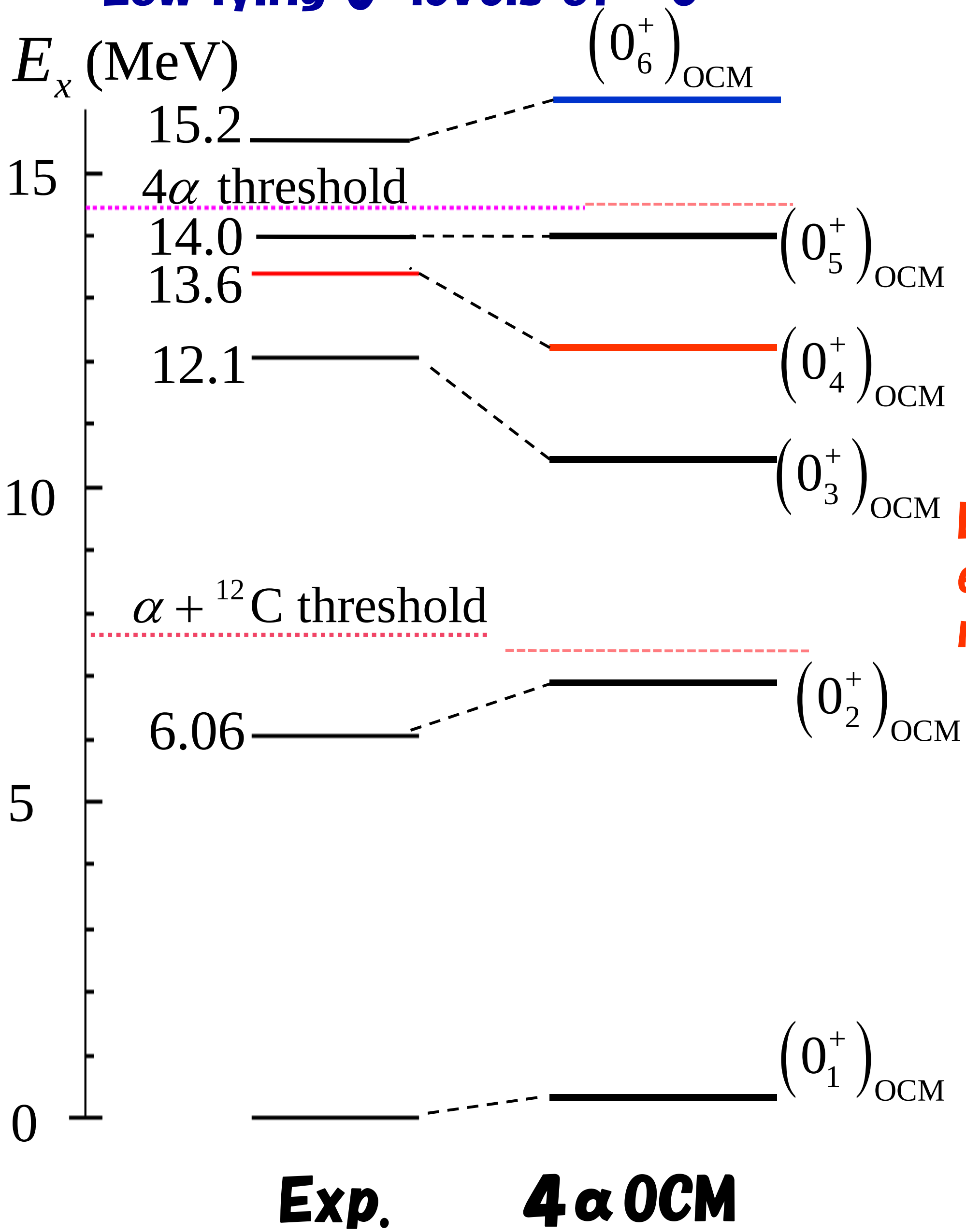
Q_{rel} : H.O. quantum number regarding α - α motion

Energies from 4α threshold

	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	−7.32	−7.28
^{12}C(2_1^+)	−4.88	−2.84
^{12}C(4_1^+)	2.06	6.43
^{12}C(0_2^+)	0.70	0.38
^{16}O(g.s.)	−14.2	−14.44

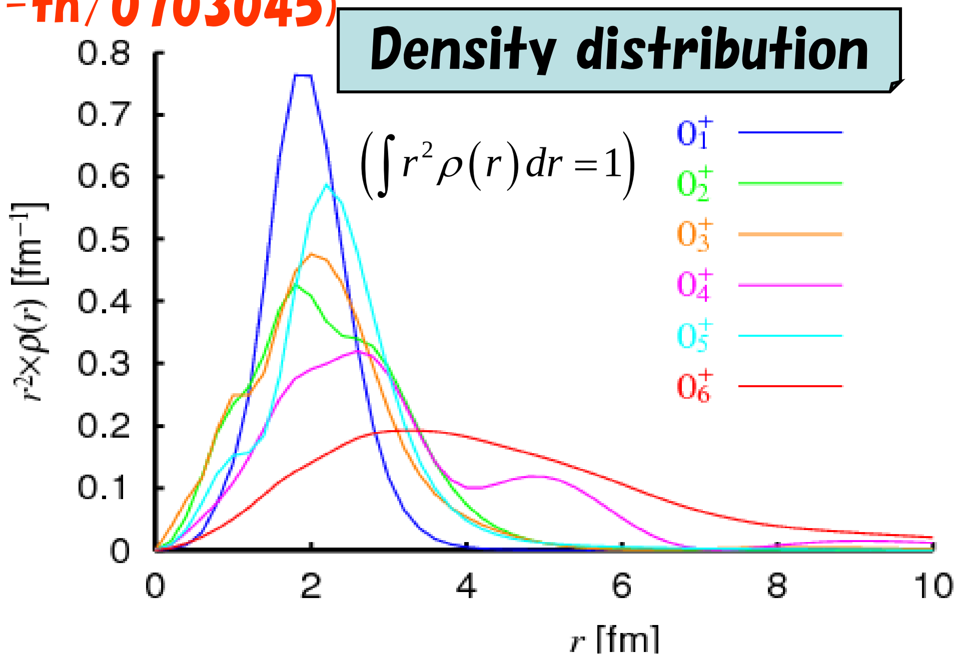
Energy levels, rms radii, monopole matrix elements and density distribution.

Low lying 0^+ levels of ^{16}O

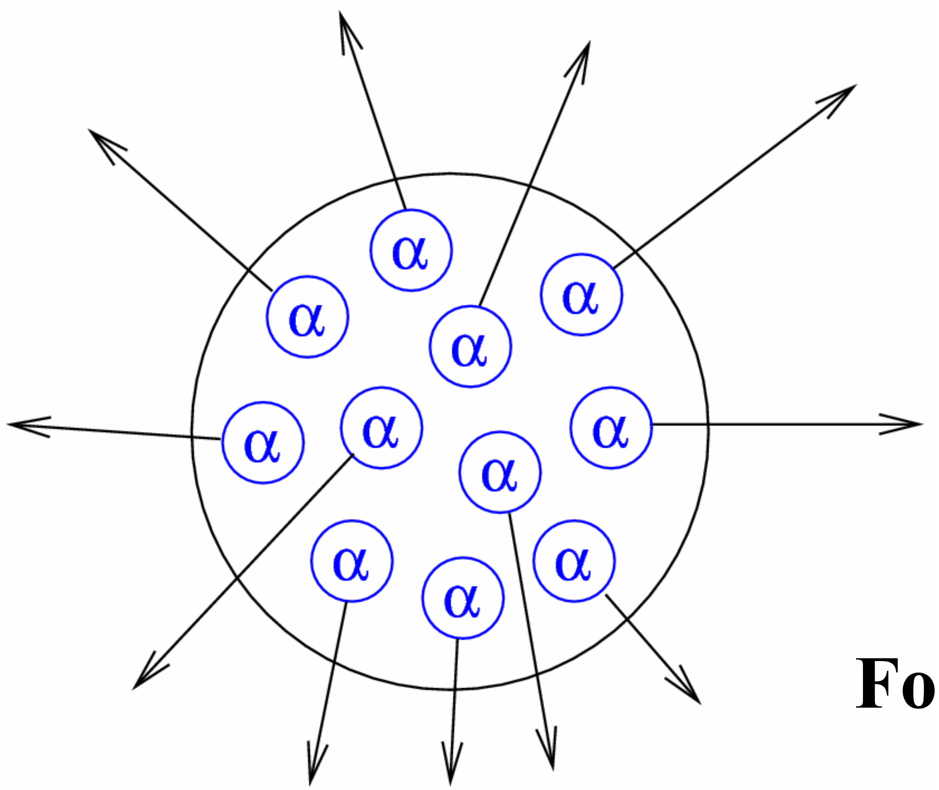


	R_{rms} (fm)	M(E0) (fm^2)	M(E0) (fm^2) Exp.
$(0_1^+)_{OCM}$	2.7		
$(0_2^+)_{OCM}$	3.1	4.2	0_2^+: 3.55
$(0_3^+)_{OCM}$	2.9	4.1	0_3^+: 4.03
$(0_4^+)_{OCM}$	3.9	2.4	0_4^+: no data
$(0_5^+)_{OCM}$	3.1	2.0	0_5^+: 3.3
$(0_6^+)_{OCM}$	5.4	1.4	0_6^+: no data

Large monopole matrix element can be the evidence of cluster states (Yamada, Y.F. et el., nucl-th/0703045)



Expanding Many α – Particle Coherent State:



All α's can be detected in coincidence

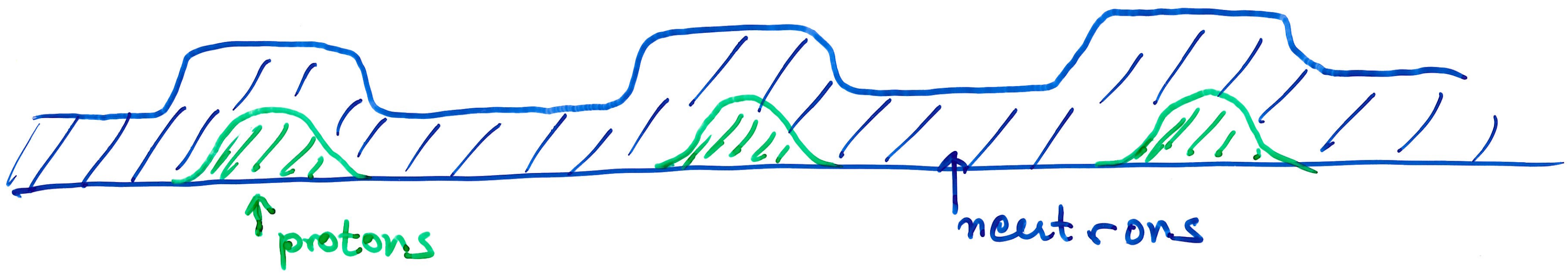
➔Coherent State Established

For example, $^{40}Ca^{*} \rightarrow 10\ \alpha$'s

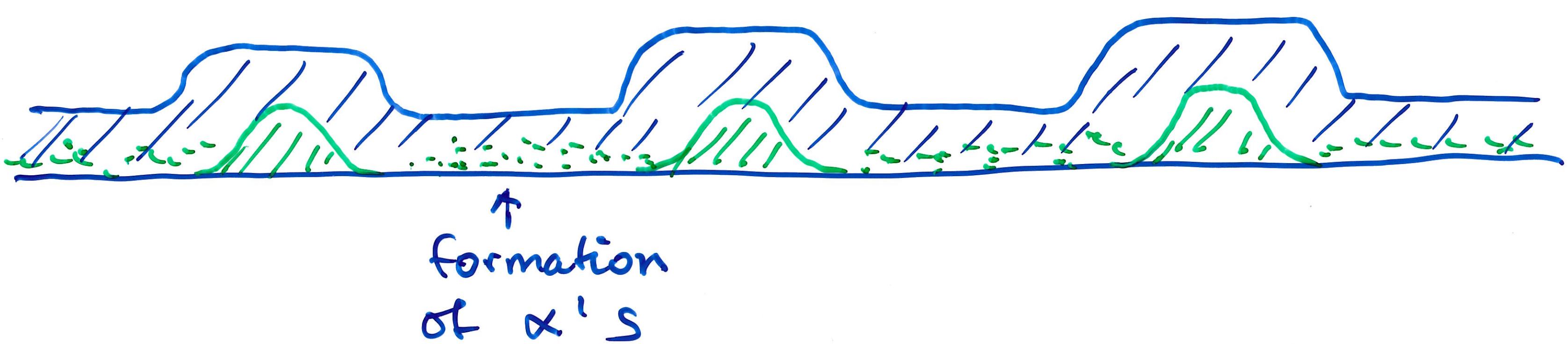
Search at Orsay

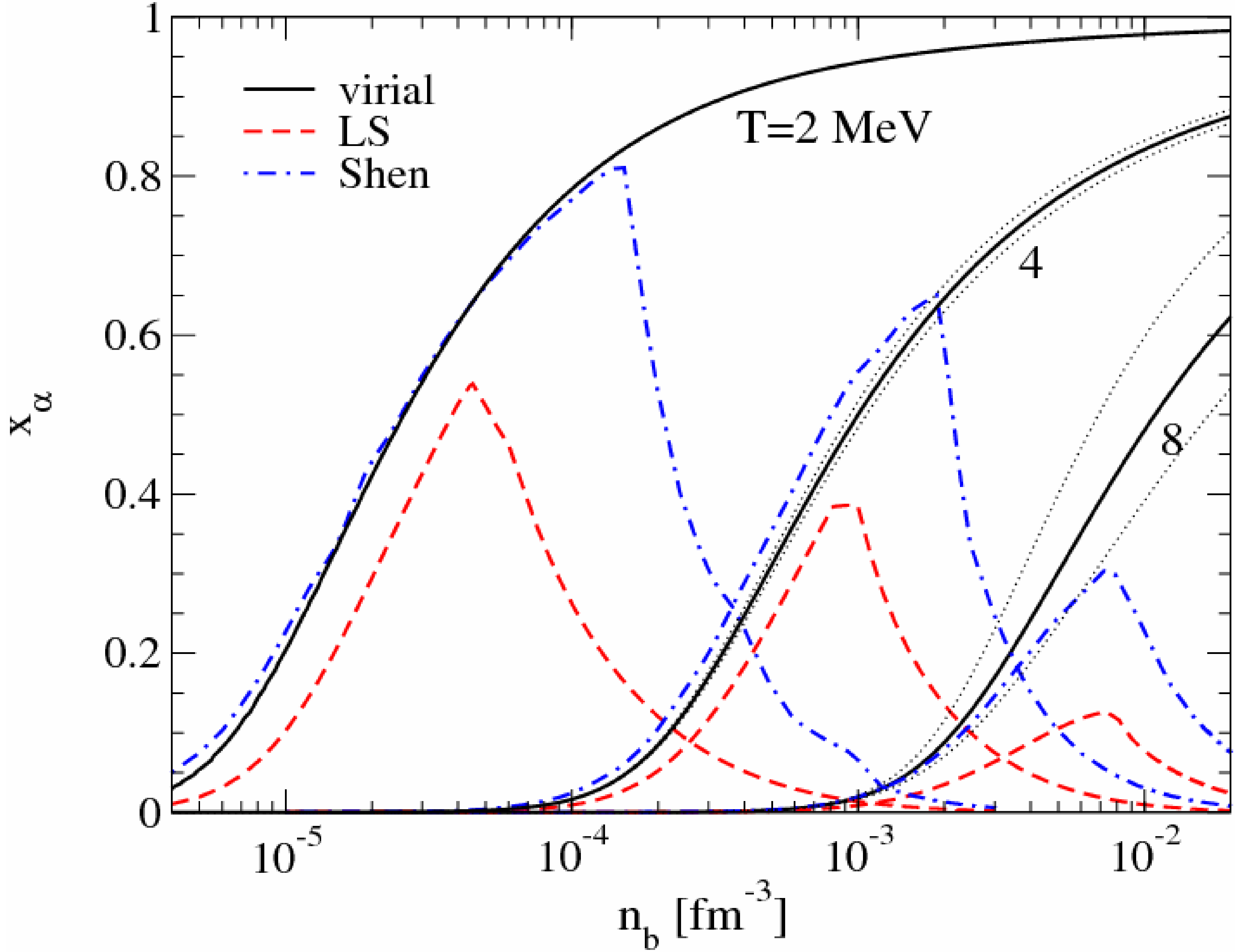
Alpha particle condensation in crust of neutron stars?

$T=0$



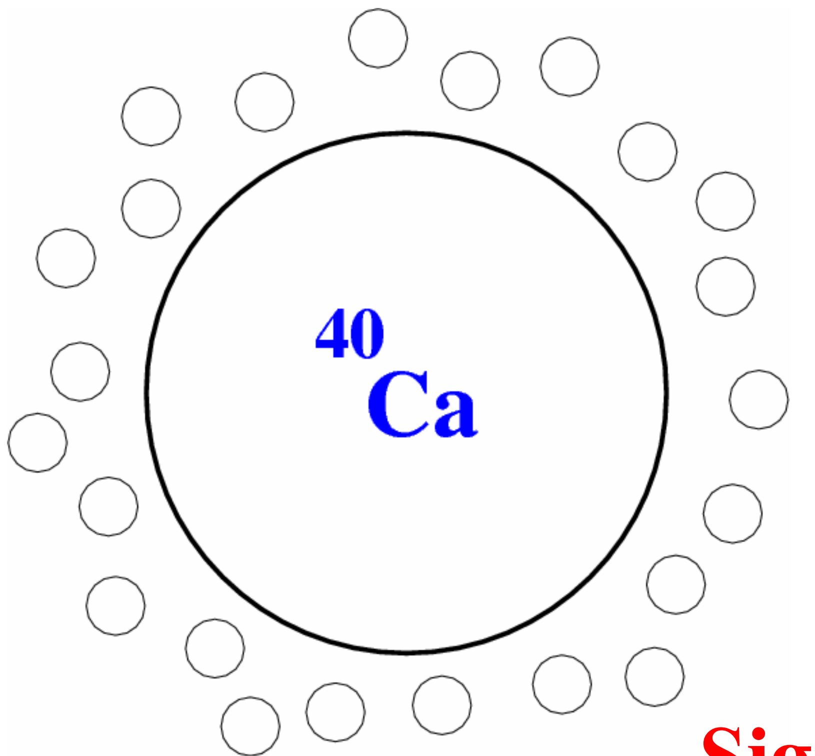
$T>0$





virial
LS
Shen
T=2 MeV
4
8
x_α
n_b [fm^{-3}]
0
0.2
0.4
0.6
0.8
1
10^{-5}
10^{-4}
10^{-3}
10^{-2}

Compound Nuclei with α – Gas:

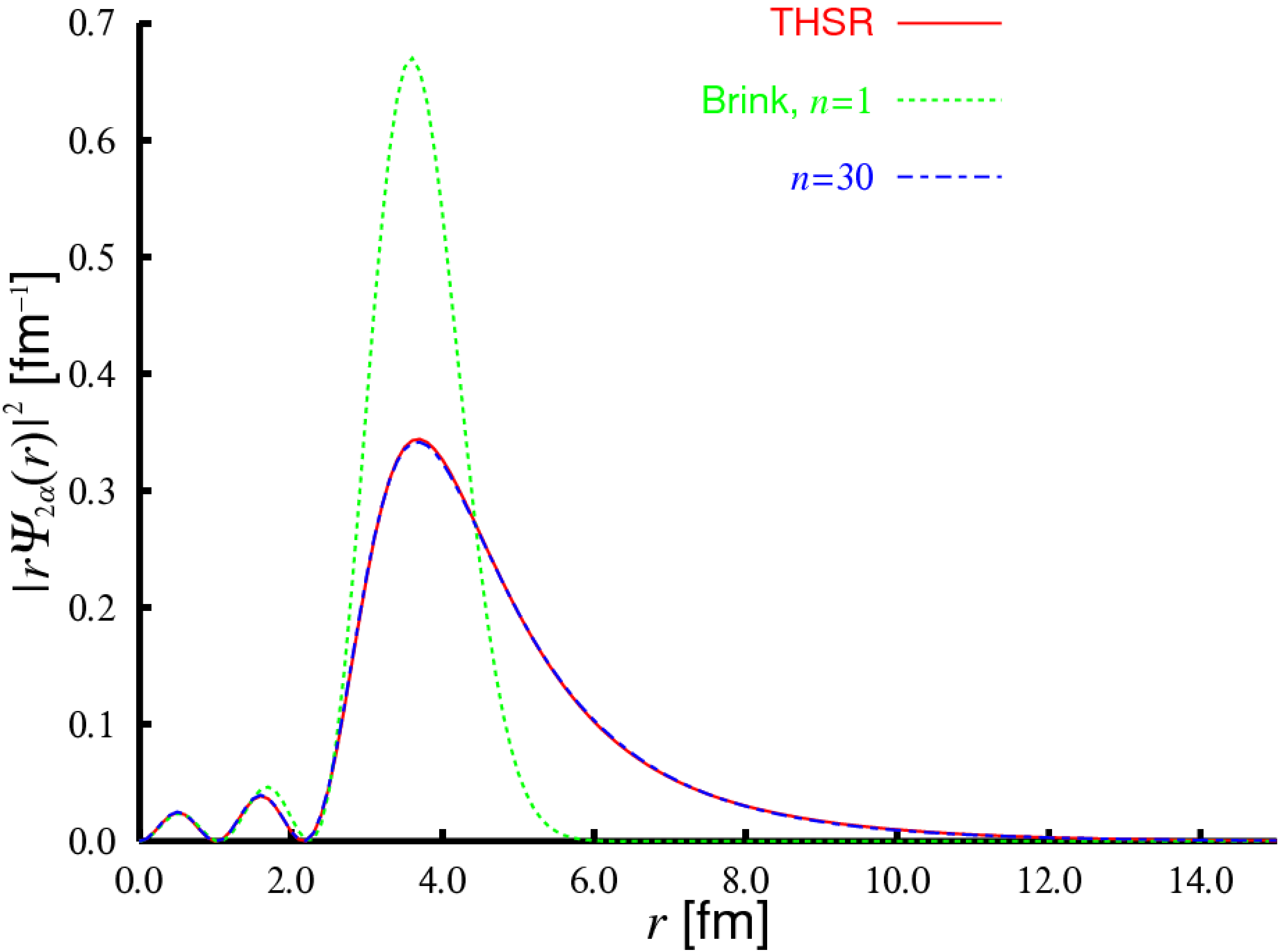


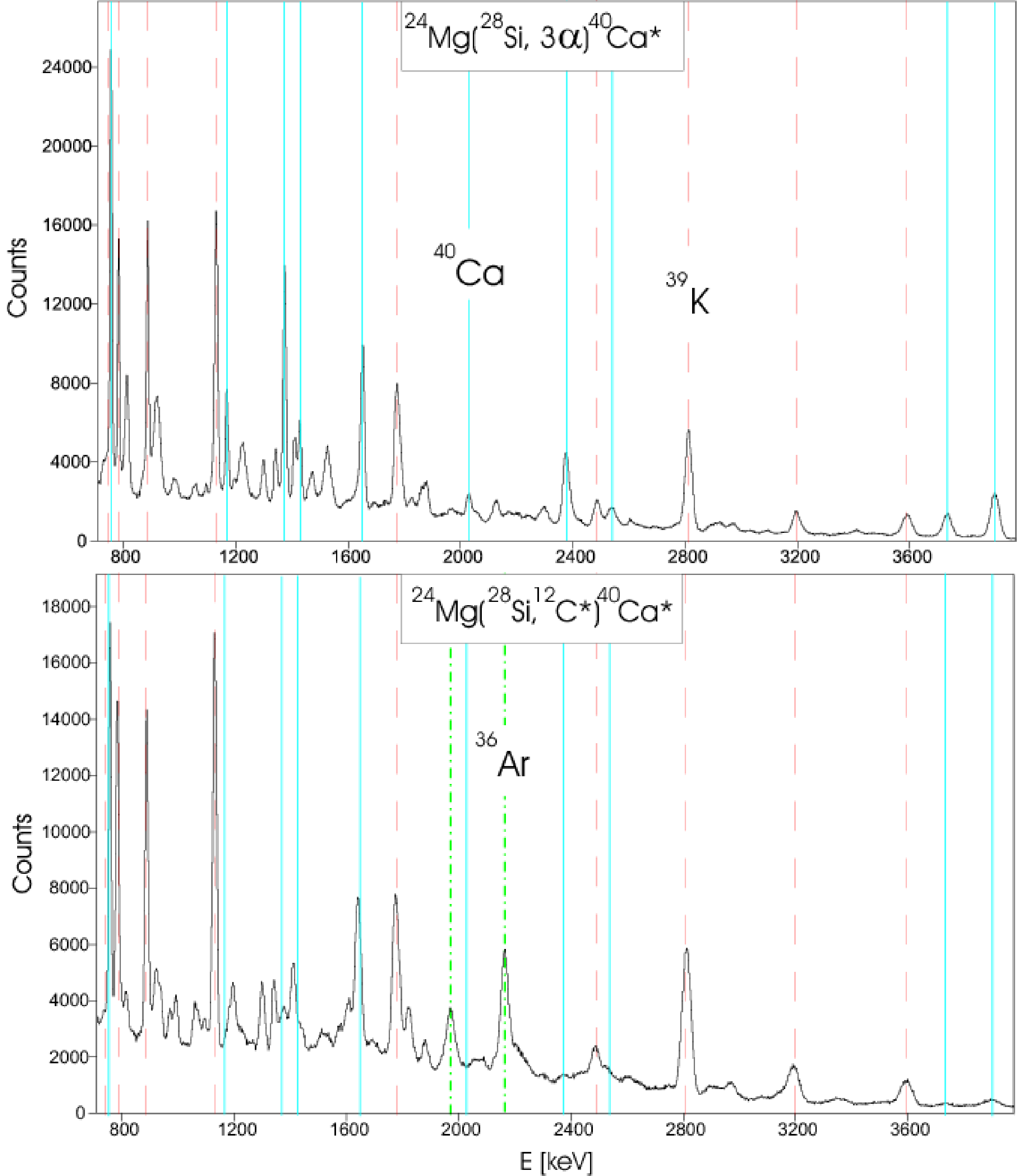
Disintegration into 2 α's

Enhanced ➔

Sign of α – Particle Condensate

(W. V. O.)


THSR
Brink, n=1
n=30
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
|rΨ2α(r)|2 [fm-1]
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
r [fm]



$^{24}Mg(^{28}Si, 3\alpha)^{40}Ca^*$
^{40}Ca
^{39}K
Counts
$^{24}Mg(^{28}Si,^{12}C^*)^{40}Ca^*$
^{36}Ar
Counts
E [keV]

Conclusions, Outlook:

- **Loosely Bound 3 – α State in $^{12}C^*$ Finally Established**
- **More α – Particle Condensates Very Likely to Exist**
- **Nuclei Unique Fermi Systems for Cluster Effects. Eventually also in Future Cold Atom Experiments**
- **May be α – Condensation Important in Proto – Neutron stars**

Collaborators:

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T. Yamada, W. von Oertzen

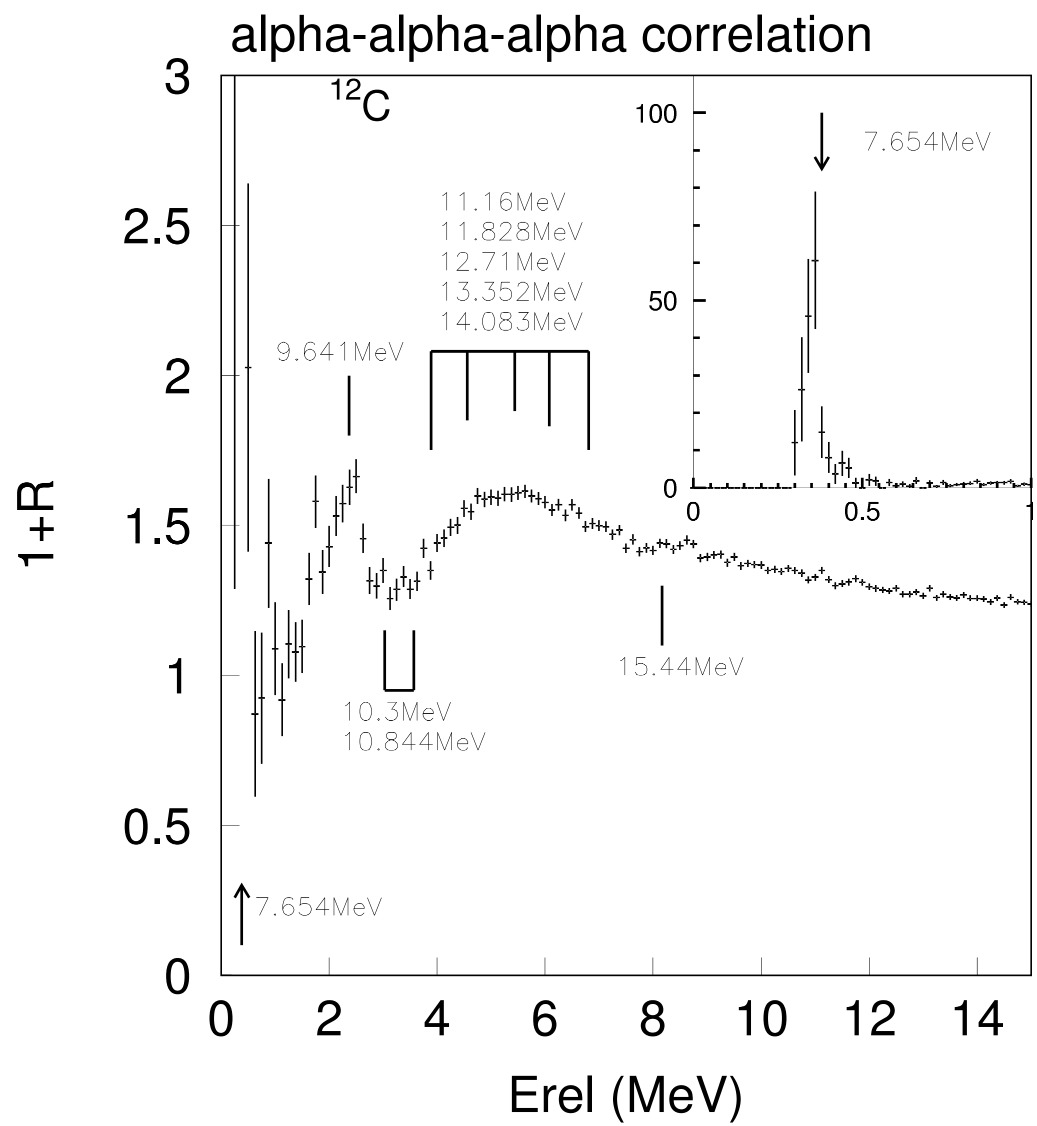
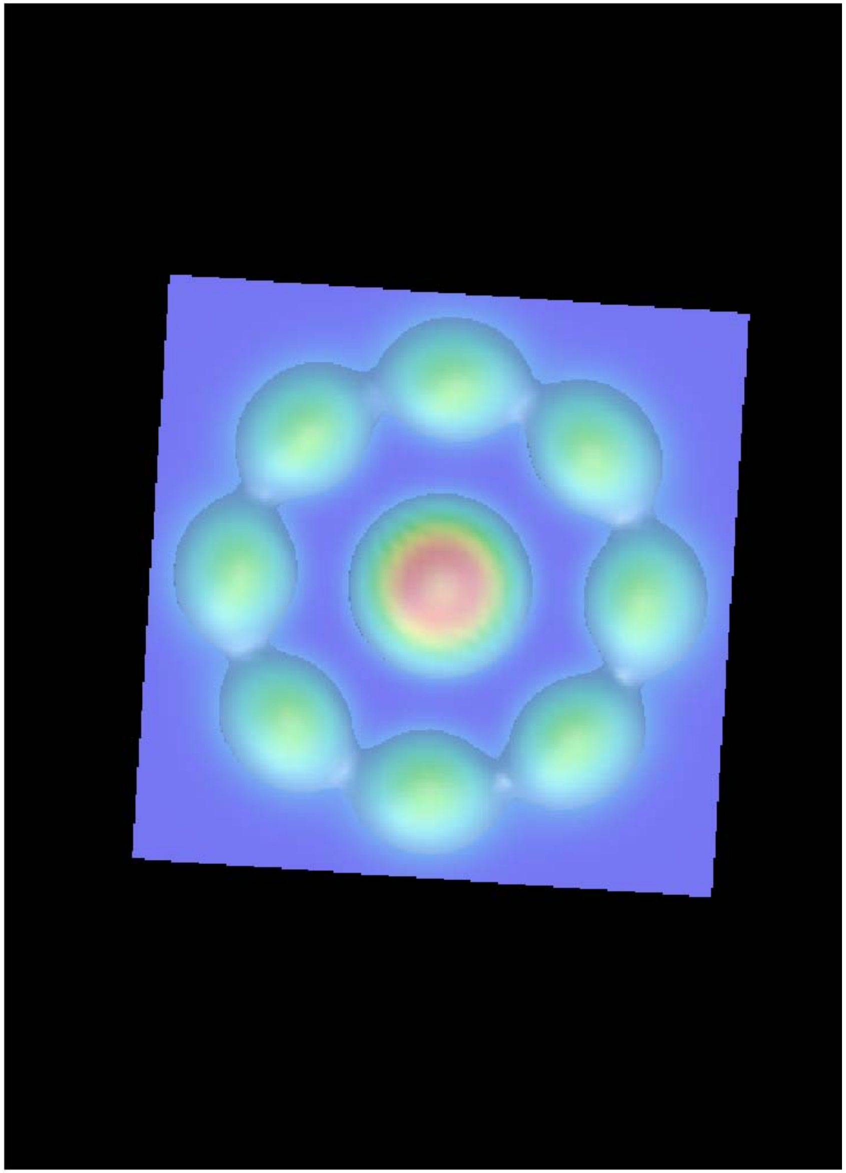
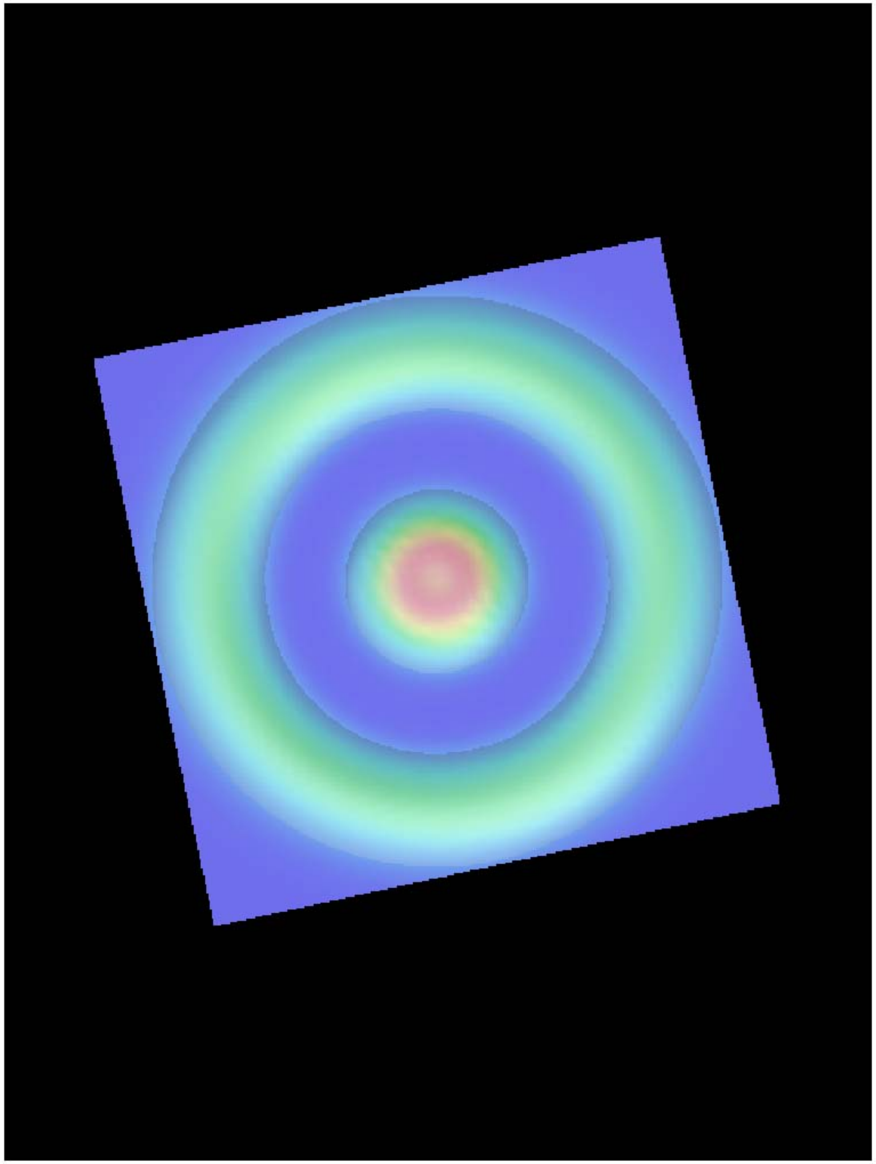
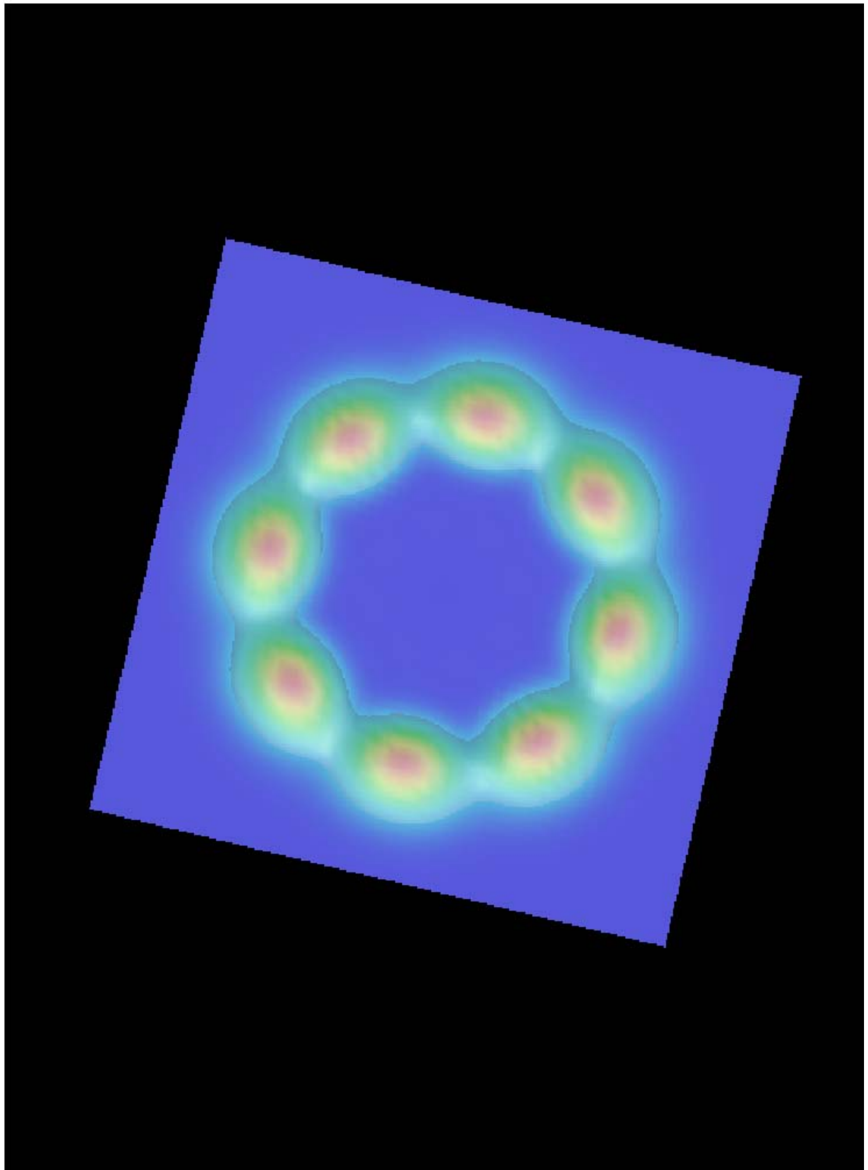
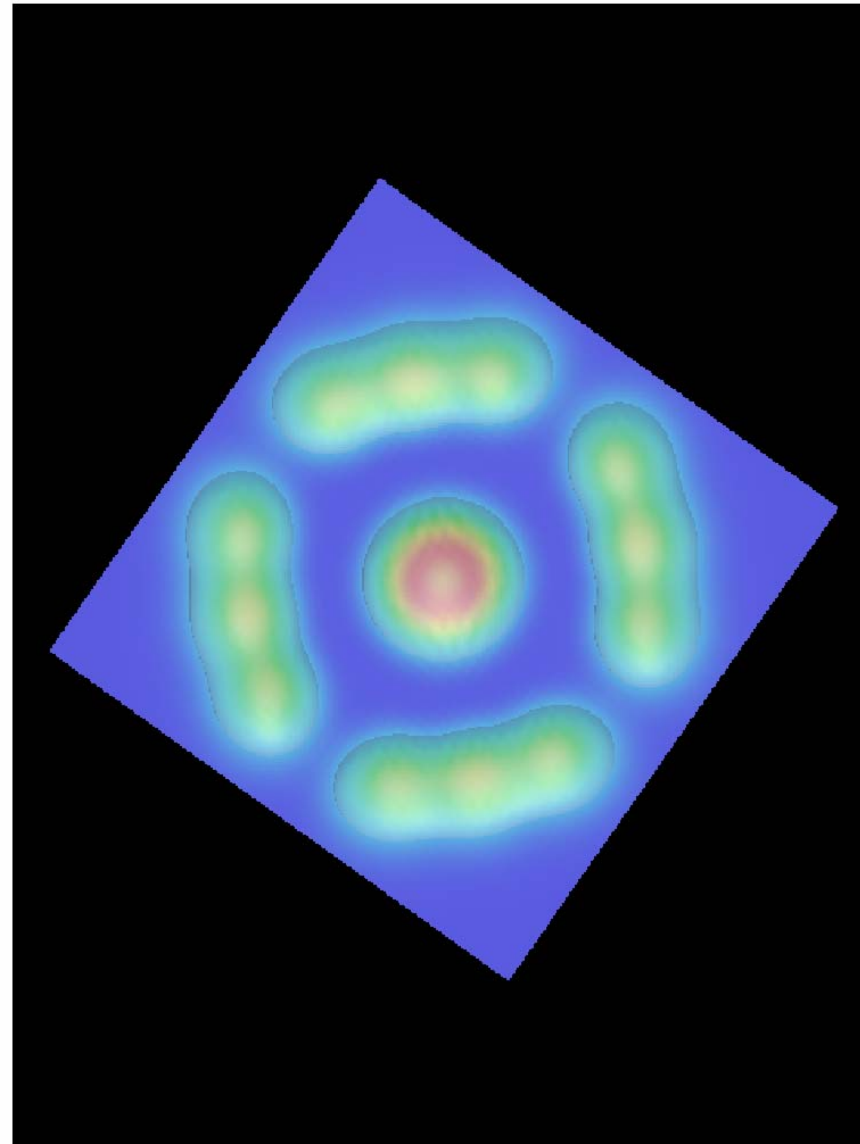


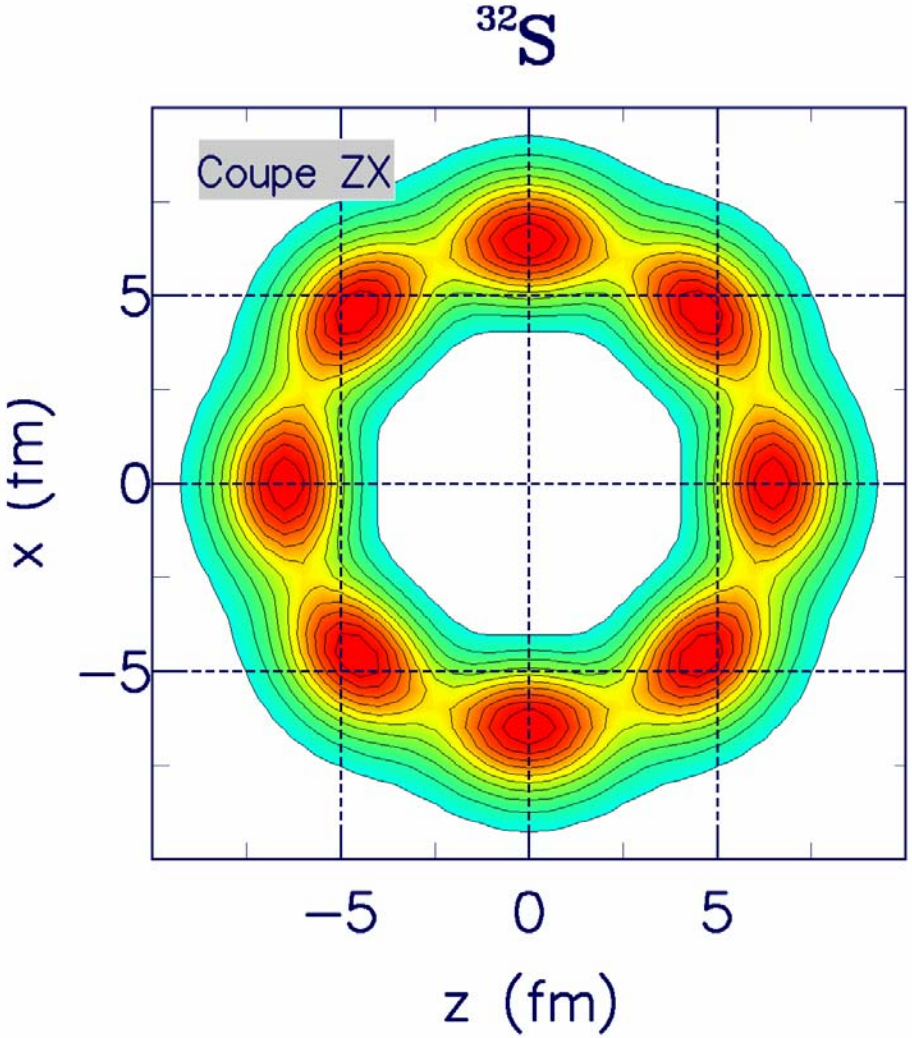
Figure 5.27: The α-α-α correlation function is shown. Resonances from the excited states of ^{12}C are labelled with the first peak seen more clearly in the inner upright panel.



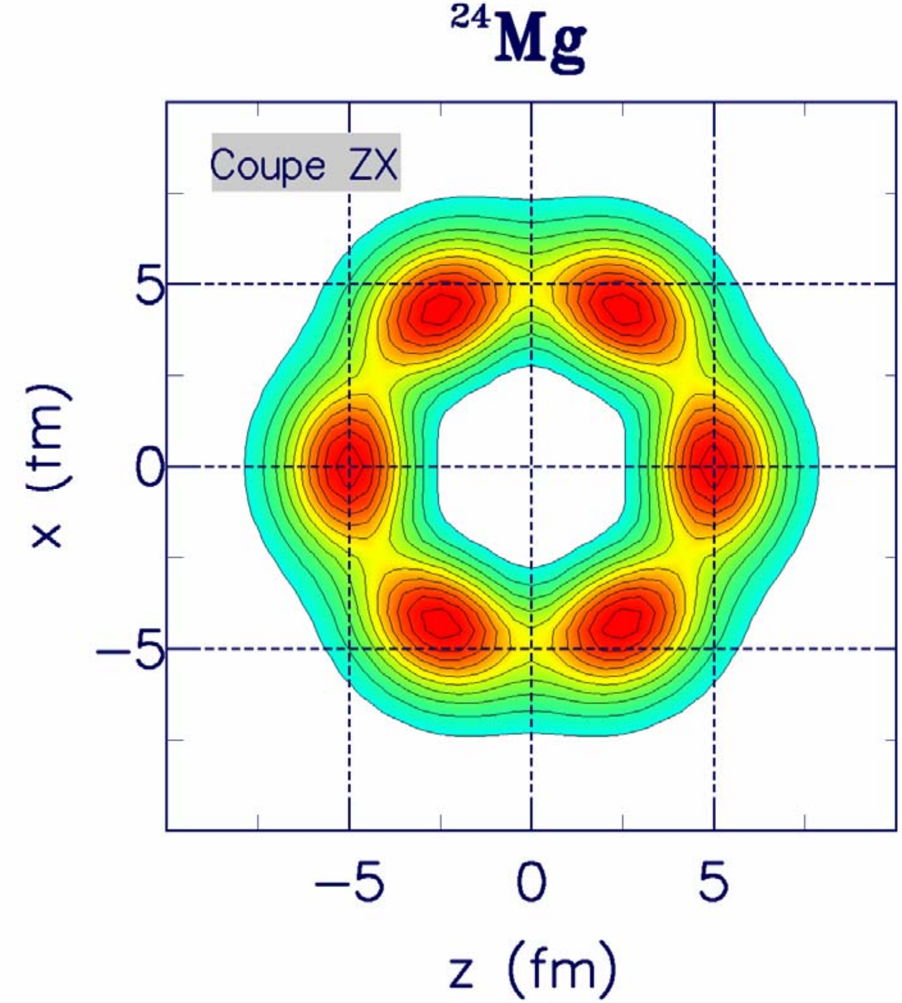




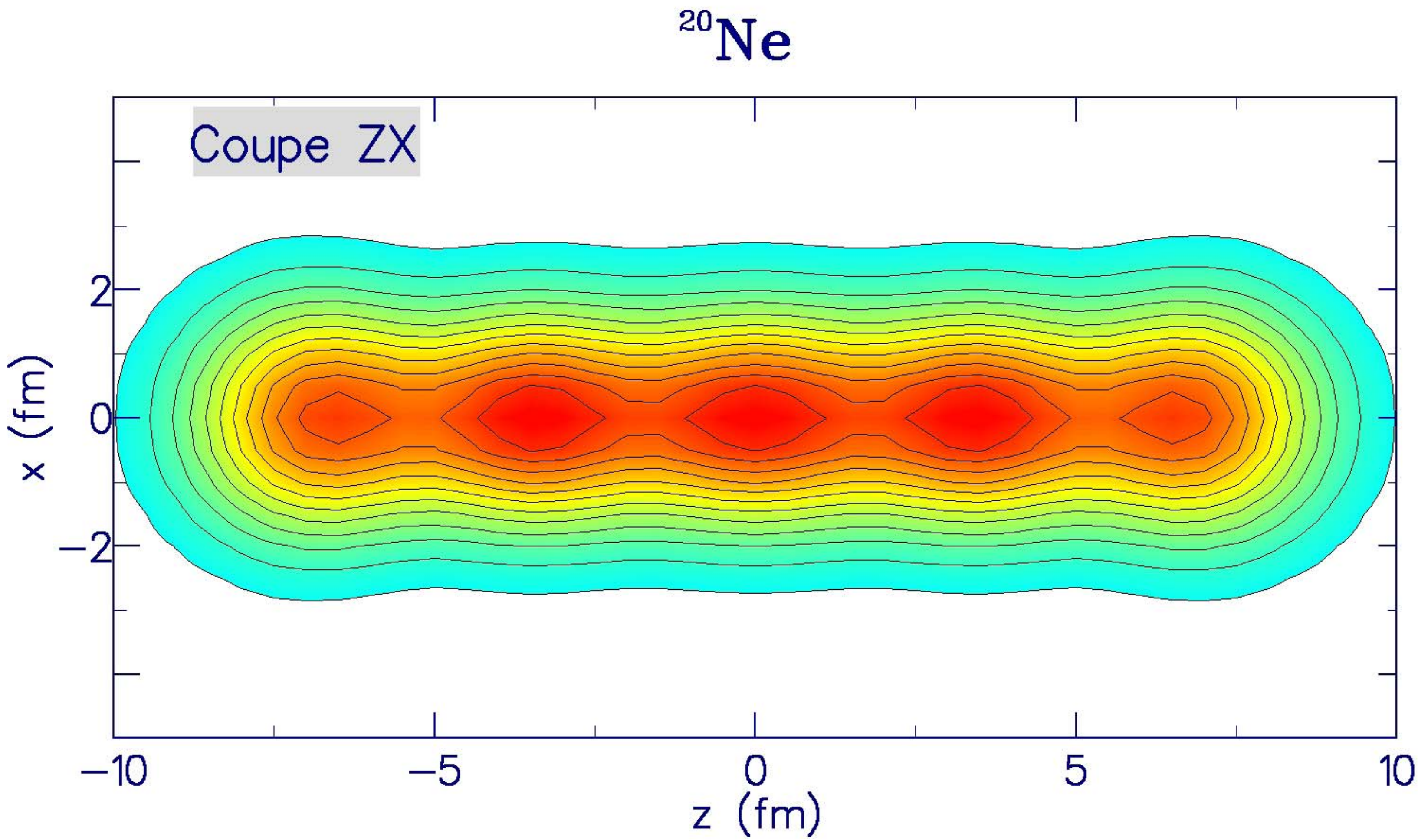




^{32}S
Coupe ZX
x (fm)
z (fm)
5
0
−5
−5
0
5



^{24}Mg
Coupe ZX
x (fm)
z (fm)
5
0
−5
−5
0
5



^{20}Ne
Coupe ZX
x (fm)
z (fm)
2
0
−2
−10
−5
0
5
10